12-1 Study Guide and Intervention (continued)

Designing a Survey

Sampling Techniques Suppose you want to survey students about their choice of radio stations. All students make up the population you want to survey. A sample is some portion of the larger group that you select to represent the entire group. A census would include all students within the population. A random sample of a population is selected so that it is representative of the entire population.

<table>
<thead>
<tr>
<th>Simple Random Sample</th>
<th>Stratified Random Sample</th>
<th>Systematic Random Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>a sample that is as likely to be chosen as another from a population</td>
<td>a population is first divided into similar, nonoverlapping groups. A sample random sample is then chosen from each group.</td>
<td>items are selected according to a specified time or interval.</td>
</tr>
</tbody>
</table>

**Example 1**

**SCHOOL** Ten students are randomly chosen from each high school class to be on an advisory committee with the principal.

- **a.** Identify the sample and suggest a population from which it was chosen.
  - The sample is 4 groups of 10 students each from the freshmen, sophomore, junior, and senior classes. The population is the entire student body of the school.

- **b.** Classify the sample as simple, stratified, or systematic.
  - This is a stratified random sample because the population was first divided into nonoverlapping groups and then a random sample was chosen from each group.

**Exercises**

Identify each sample, suggest a population from which it was selected. Then classify the sample as simple, stratified, or systematic. Explain your reasoning.

1. **SCHOOL** Each student in a class of 25 students was given a number at the beginning of the year. Periodically, the teacher chooses 4 numbers at random to display their homework on a projector.
   - **4 students; 25 students in the class; simple**

2. **GARDENING** A gardener divided a lot into 25-square-foot sections. He then took 2 soil samples from each and tested the samples for mineral content.
   - **2 soil samples from each section; entire lot; stratified**

3. **SCHOOL** One hundred students in the lunch room are chosen for a survey. All students in the school eat lunch at the same time. 100 students; all students; simple

4. **SHOPPING** Every tenth person leaving a grocery store was asked if they would participate in a community survey.
   - **Every tenth person leaving a grocery store; all shoppers at the grocery store; systematic**

**Example 2**

**DOOR PRIZES** Each of the participants in a conference was given a numbered name tag. Twenty-five numbers were chosen at random to receive a door prize.

- **a.** Identify the sample and suggest a population from which it was chosen.
  - The sample was 25 participants of the conference. The population was all of the participants of the conference.

- **b.** Classify the sample as simple, stratified, or systematic.
  - Since the numbers were chosen randomly, this is a simple random sample because each participant was equally likely to be chosen.

**Exercises**

Identify each sample as biased or unbiased. Explain your reasoning.

2. **MANUFACTURING** A company that produces motherboards for computers randomly selects 25 board motherboards out of a shipment of 1500, and then tests each selected motherboard to see that it meets specifications. Unbiased; the sample is representative and randomly chosen.

5. **GOVERNMENT** The first 100 people entering a county park on Thursday are asked their opinions on a proposed county ordinance that would allow dogs in county parks to go unleashed in certain designated areas. Biased; the convenience sample may not accurately reflect the population.

6. **MUSIC** To determine the music preferences of their customers, the owners of a music store randomly choose 10 customers to participate in an in-store interview in which they listen to new CDs from artists in all music categories.
   - **10 music-store customers; all customers in a music store; simple; sample is equally likely to be chosen as any other sample from the population**

7. **LIBRARIES** A community library asks every tenth patron who enters the library to display their homework on a projector.
   - **A group of library patrons; all patrons of a library; systematic; individuals are selected according to a specified interval**

8. **COMPUTERS** To determine the number of students who use computers at home, the high school office chooses 10 students at random from each grade, and then interviews the students.
   - **10 students from each grade of a high school; all students in the high school; stratified; the population is first divided into similar, nonoverlapping groups**

**Skills Practice**

1. **LANDSCAPING** A homeowner is concerned about the quality of the topsoil in her back yard. The back yard is divided into 5 equal sections, and then a 1-inch plug of topsoil is randomly removed from each of the 5 sections. The soil is taken to a nursery and analyzed for mineral content.
   - **5 one-inch plugs of topsoil; all the topsoil in the back yard; stratified**

2. **HEALTH** A hospital’s administration is interested in opening a gym on the premises for all its employees. They ask each member of the night-shift emergency room staff if he or she would use the gym, and if so, what hours the employee would prefer to use it.
   - **The night-shift emergency room staff at a hospital; all employees at the hospital; voluntary response**
1. GOVERNMENT At a town council meeting, the chair asks 5 citizens attending for their opinions on whether to approve rezoning for a residential area. 5 citizens of a town; all citizens of a town; biased; convenience

2. BOTANY To determine the extent of leaf blight in the maple trees at a nature preserve, a botanist divides the reserve into 10 sections, randomly selects a 200-foot by 200-foot square in the section, and then examines all the maple trees in the section. The maple trees in a square area of each of 10 sections at a nature preserve; all the maple trees at the nature preserve; unbiased; stratified

3. FINANCE To determine the popularity of online banking in the United States, a polling company sends a mail-in survey to 5000 adults to see if they bank online, and if they do, how many times they bank online each month. 5000 U.S. adults; all U.S. adults; biased; voluntary response

4. SHOES A shoe manufacturer wants to check the quality of its shoes. Every twenty minutes, 20 pairs of shoes are pulled off the assembly line for a quality inspection. unbiased; the sample is systematically chosen

5. BUSINESS To learn which benefits employees at a large company think are most important, the management has a computer select 50 employees at random. The employees are then interviewed by the Human Relations department. unbiased; the sample is random

For Question 6, identify the sample, and suggest a population from which it was selected. Then classify the type of data collection used.

6. BUSINESS An insurance company checks every hundredth claim payment to ensure that claims have been processed correctly. Every hundredth claim payment at an insurance company; all claim payments at an insurance company; systematic; the sample was selected according to a specified interval

7. ENVIRONMENT Suppose you want to know if a manufacturing plant is discharging contaminants into a local river. Describe an unbiased way in which you could check the river water for contaminants. A different time each day, take a 10-ounce sample of water from given locations just upstream and just downstream from where the plant discharges its wastes. Compare the samples for contaminants to see if any are entering the river from the discharge.

8. SCHOOL Suppose you want to know the issues most important to teachers at your school. Describe an unbiased way in which you could conduct your survey. Sample answer: Obtain a list of all teachers at the school. Assign each teacher a number, and then randomly select 10 numbers. Interview each of the teachers assigned one of the selected numbers.

5. CHILD SAFETY The British Columbia Automobile Association performed a free child safety seat inspection for people that came in for the safety check. Only 7% of the 1000 seats inspected were properly used. The graph below shows the approximate percentages of results for the safety seat inspections.

Child Safety Seat Survey Results

- Child in incorrect seat for age/size/weight: 3%
- Child seat not tightly secured in vehicle: 44%
- Seat properly used: 7%
- Tethers not used: 14%
- Incorrect placement of shoulder harness: 9%

Source: Autonet

a. Write a statement to describe the sampling technique. The sample is biased because it was based on voluntary response.

b. Is it appropriate to say that 23% of children in child safety seats in British Columbia are not securely fastened in their seats? Explain. No. This was not a random sample, so the data cannot be generalized to the population. We do not know how the seats are being used in vehicles that were not brought in.
Heads or Tails

Based on the way certain coins are manufactured, there is a 'bias' towards landing either heads up or tails up when they are flipped. This can be demonstrated by dropping a bag of pennies from the same minting year on their edges on a table. Gently shake the table so that the pennies fall flat.

1. What percent of the pennies would you expect to land heads up?
   Sample answer: 50%

2. What percent actually landed heads up?
   See students' work, but it should be more heads than tails.

3. Repeat the experiment.
   See students' work. They should be similar.

4. Make a conjecture as to why the percent of pennies that land face up differs from what you would expect it to be.
   Sample answer: more weight on one side, grooves in the penny.

5. When a penny is manufactured, it is beveled or grooved slightly, so that when it falls, it is more likely to fall heads up. There appears to be a difference between minting years. Repeat this experiment with other minting years.

6. Are the results about the same as before? Explain why or why not.
   See students' work. Because of this bias, tossing a coin and letting it hit the ground or a table to determine a winner is not fair. The penny is most likely to land with the head side up.

7. How can you change a coin toss so that heads and tails are more equally likely to appear?
   Sample answer: Catch it before it hits the ground; find a balanced coin. Try this experiment with other coins to determine if there is a "bias" in manufacturing these coins as well.

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### Exercises

1. RESTAURANTS
   A restaurant records the number of people who order soup at lunch each day: {26, 25, 30, 32, 27, 28, 28, 28, 29, 30, 28}.
   List the values from least to greatest: 25, 26, 27, 28, 28, 28, 29, 30, 30. The data set does not have any outliers, but does have many repeated numbers. The mean would best represent the data.
   The mean of the data is 28.

2. SOCCER
   A soccer team keeps a record of the number of points it scores in each game: {2, 3, 2, 1, 4, 3, 1, 3, 3, 4}.
   List the values from least to greatest: 1, 1, 2, 2, 3, 3, 3, 3, 4, 4. The data set has four sets of repeated numbers. The mode best represents the data. The mode is 3, the number that occurs the most often.

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### Lessons

#### Analyzing Survey Results

To make survey data more useful, it can be summarized according to measures of central tendency: mean, median, and mode.

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Most Used When</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>the sum of the data divided by the number of items in the data set</td>
<td>The data sets have no outliers.</td>
</tr>
<tr>
<td>median</td>
<td>the middle number of the ordered data, or the mean of the middle two numbers</td>
<td>The data sets have no outliers, but there are no big gaps in the middle of the data.</td>
</tr>
<tr>
<td>mode</td>
<td>the number or numbers that occur the most often</td>
<td>The data set has many repeated numbers.</td>
</tr>
</tbody>
</table>

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#### Example

<table>
<thead>
<tr>
<th>Type</th>
<th>Which measure of central tendency best represents the data?</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESTAURANTS: {26, 25, 30, 32, 27, 28, 28, 28, 29, 30, 28}</td>
<td>Mean, approximately 28</td>
</tr>
<tr>
<td>SOCCER: {2, 3, 2, 1, 4, 3, 1, 3, 3, 4}</td>
<td>Mode, 3</td>
</tr>
</tbody>
</table>

#### Exercises

1. DEFECTS
   A furniture manufacturer keeps records of how many units are defective each day: {7, 12, 9, 8, 10, 14, 8}.
   List the values from least to greatest: 7, 8, 8, 9, 10, 12, 14. The data set has no outliers, but does have many repeated numbers. The mean would best represent the data.
   The mean of the data is 9.7.

2. SCIENCE TESTS
   Mr. Wharton records his students' scores on the last science test: {94, 88, 88, 94, 94, 84, 94}.
   List the values from least to greatest: 84, 84, 88, 88, 94, 94, 94. The data set has many repeated numbers. The mode best represents the data. The mode is 94, the number that occurs the most often.

3. PUPPIES
   A veterinarian keeps records of the weights of puppies in ounces: {4.1, 3.8, 5.0, 4.6, 5.6, 4.7, 11.6}.
   List the values from least to greatest: 3.8, 4.1, 4.6, 4.7, 5.0, 5.6, 11.6. There are no outliers. The mean best represents the data. The mean of the data is 5.52.

4. COMMUTING
   The local newspaper conducted a telephone survey of commuters to see how they get to work each day. The responses were: commuter rail, 22; bus, 17; subway, 18; walking, 15; car, 224.
   List the values from least to greatest: 15, 17, 18, 22, 224. The mode is car, the number that occurs the most often.

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### Study Guide and Intervention

#### Analyzing Survey Results

To make survey data more useful, it can be summarized according to measures of central tendency: mean, median, and mode.

- **Mean**: The sum of the data divided by the number of items in the data set.
- **Median**: The middle number of the ordered data, or the mean of the middle two numbers.
- **Mode**: The number or numbers that occur the most often.

#### Exercises

1. RESTAURANTS
   A restaurant records the number of people who order soup at lunch each day: {26, 25, 30, 32, 27, 28, 28, 28, 29, 30, 28}.
   List the values from least to greatest: 25, 26, 27, 28, 28, 28, 29, 30, 30. The data set does not have any outliers, but does have many repeated numbers. The mean would best represent the data.
   The mean of the data is 28.

2. SOCCER
   A soccer team keeps a record of the number of points it scores in each game: {2, 3, 2, 1, 4, 3, 1, 3, 3, 4}.
   List the values from least to greatest: 1, 1, 2, 2, 3, 3, 3, 3, 4, 4. The data set has many repeated numbers. The mode best represents the data. The mode is 3, the number that occurs the most often.

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### Chapter 12 Glencoe Algebra 1

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### Answers

#### Lesson 12-1 and Lesson 12-2

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### Glencoe Algebra 1
**12-2 Study Guide and Intervention (continued)**

### Analyzing Survey Results

**Evaluate Survey Results**

After a survey's data has been summarized and a report of the findings and conclusions has been made, it is important to be able to judge the reliability of the report. You can do this by verifying that the sample is truly random, that the sample is large enough to be an accurate representative of the population, and that the source of the data is a reliable one. Also check graphs accompanying surveys for misleading results.

**Example**

**MUSIC**  
Given the following portion of a survey report, evaluate the validity of the information and conclusion.

**Question:** What is your favorite band?  
**Sample:** 100 concertgoers were randomly selected.  
**Conclusion:** America's favorite band is October Hope.

<table>
<thead>
<tr>
<th>Choice</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>October Hope</td>
<td>40%</td>
</tr>
<tr>
<td>Rayne</td>
<td>20%</td>
</tr>
<tr>
<td>Weimar Republic</td>
<td>10%</td>
</tr>
<tr>
<td>Larry Blodgett Trio</td>
<td>30%</td>
</tr>
</tbody>
</table>

Source: October Hope Fan Club

The report says that concertgoers were chosen randomly, but there is no guarantee that a group of concertgoers is representative of America as a whole. In addition, a sample size of 100 may be too small to draw a conclusion from.

Also, the report's source is the “October Hope Fan Club,” which may be biased, considering that the report cites October Hope as America's favorite band.

### Exercises

**Given the following portion of a survey report, evaluate the validity of the information and conclusion.**

1. **SCHOOL UNIFORMS**  
   Survey USA polled 500 randomly selected adults in Cincinnati, Ohio, by telephone.
   **Question:** Should public school students wear uniforms?  
   **Results:** 58% said yes, 32% said no, 10% said unclear.  
   **Conclusion:** Adults in Cincinnati believe students should wear uniforms to school.

2. **ELECTIONS**  
   State Representative Beck commissioned a poll of 400 randomly selected adults visiting a mall in her district.
   **Question:** Do you approve of the job State Representative Beck is doing?  
   **Results:** 250 approved, 150 did not approve, 30 undecided.  
   **Conclusion:** Senator Beck will win re-election.

### Determine whether each display gives an accurate picture of the survey results.

1. **TRASH INCINERATORS**  
   A local newspaper surveyed 530 randomly chosen Eastwich residents.
   **Question:** Do you support closing the trash incinerator in Eastwich?  
   **Results:** 300 said yes, 200 said no, 10 undecided.  
   **Conclusion:** Eastwich residents overwhelmingly support closing the trash incinerator. The graph makes it appear as if there were twice as many “Yes” responses as “No” ones, but there were only 310 “Yes” responses compared to 220 for “No.”

2. **ISSUES**  
   A television station interviewed 400 randomly chosen Eastwich residents.
   **Question:** What issue matters most to you in choosing a candidate to vote for?  
   **Results:** Environment 63%, War 24%, Social Issues 9%, Economy 4%.  
   **Conclusion:** Voters care about the environment, but it does not necessarily support the conclusion. Voters may care about the environment, but care about another issue more.
12-2 Practice
Analyzing Survey Results

Which measure of central tendency best represents the data? Justify your answer. Then find the measure.

1. CALCULATORS The math department counts how many graphing calculators are in each classroom: {20, 19, 20, 20, 18, 19, 20, 18, 19}. mode; 20
2. BUDGETING The Brady family keeps track of its monthly electric bills: $134, $122, $128, $136, $120, $129. mean; $128
3. AUTOMATED TELLERS A bank keeps track of how many customers use its ATM each hour: {39, 42, 4, 120, 54, 48, 43}. median; 44

Given the following portion of a survey report, evaluate the validity of the information and conclusion.

4. HOMEWORK Chris polled 16 of his friends during study hall. Question: Do teachers at Edison High School assign too much homework? Results: yes, 94%; no, 6% Conclusion: Teachers at Edison High School should assign less homework.

5. SMOKING SurveyUSA polled 500 randomly selected adults in Kentucky. Question: Do you want to see smoking banned from restaurants, bars, and most indoor public places in Kentucky? Results: banned, 58%; allowed, 41%; not sure, 1% Conclusion: The United States should ban smoking indoors.

6. REDEVELOPMENT A local news broadcast commissioned a poll of 600 randomly chosen Providence residents. Question: Do you support or oppose the redevelopment of the waterfront? Conclusion: Providence residents support redeveloping the waterfront.

7. PETS Ernesto took a poll of randomly selected students at his high school and asked them how many pets they owned. He recorded the results and made the graph shown at the right. Write a valid conclusion using data to support your answer. Answers will vary.

12-2 Word Problem Practice
Analyzing Survey Results

1. PROPERTY TAXES A landlord is keeping track of what he pays each month in property taxes so he can budget accordingly. For the first half of the year, the tax bills were $256, $256, $274, $256, $256, and $274. Which measure of central tendency best represents the data? Justify your answer. Then find the measure.

2. GAS PRICES Quinnipiac University surveyed 1534 randomly chosen registered voters nationwide and asked them, “As a result of the recent rise in gas prices, have you cut back significantly on how much you drive?” Among those who made less than $30,000 a year, 67% said they had cut back on how much they drive while 30% said they had not. Based on this information, a newspaper made the conclusion that “Americans are cutting back on their driving because of high gas prices.” Evaluate the validity of the information and conclusion.

3. BODYBUILDING A bodybuilder keeps track of how many sets of each exercise he performs each day: {9, 8, 6, 5, 11, 7, 10}. Which measure of central tendency best represents the data? Justify your answer. Then find the measure.

4. TRANSPORTATION The Ford Township School Board surveyed 86 randomly selected students to find out how students get to school each day. Question: What mode of transportation did you use to get to school today? Conclusion: Most students take the bus to school every day.

Waterfront Redevelopment

Support: 10
Oppose: 5
Strongly Support: 15
Strongly Oppose: 20
Undecided: 20

School Transportation

Bus: 10
Walk: 50
Drive: 30

Answers will vary.
A7

Statistics and Parameters

Study Guide and Intervention

Chapter 12

NAME

DATE

PERIOD

12-2 Enrichment

Margin of Error

When conducting a survey, it is important to know how accurate your data are. You can learn more about the accuracy of a survey by calculating the margin of error, or the maximum statistical amount of the survey's uncertainty. When calculating the margin of error, you must first define the confidence level. The confidence level tells how accurate the percentages given in the poll are, assuming the only error present is statistical error (i.e., no bias). Most polls use a confidence level of 95%, meaning that there is at least a 95% confidence that the "true" percentage will be within plus or minus the margin of error of the percentage given in the survey.

Example

A7

Electronics

A poll of 300 randomly selected registered voters shows that the incumbent mayor of New Castle is leading her opponent by a 52% to 48% margin. Find the margin of error for the poll assuming a 95% confidence level.

Margin of Error (95% confidence level) = ± \( \frac{0.98 \sqrt{300}}{300} \)

Margin of Error (95% confidence level) = ± 0.057

The margin of error assuming a 95% confidence level is 0.057, or ±5.7%. This means the mayor’s actual support is 95% likely to be between 46.3% and 57.7%.

Exercises

1. Toll Hikes

A poll of 600 randomly selected adults shows that 65% of those surveyed are against a plan to raise bridge tolls. Find the poll’s margin of error for the 90% confidence level ± 3.3%

2. Vacations

A poll of 1100 randomly selected adults shows that 34% of those surveyed are planning to visit the beach this summer. Find the poll’s margin of error for the 90% confidence level ± 3.1%

3. Politics

A poll of 785 randomly selected registered voters shows that 47% of voters will vote to re-elect the incumbent state senator.

a. Find the margin of error for the 95% confidence level ± 4.0%

b. Express the range of possible support for the state senator as an inequality for the 95% confidence level: 43.5% ≤ x ≤ 50.5%

4. about how many people would need to be surveyed to give a margin of error of ± 1% for the 95% confidence level? 9604

Example

Elections

A poll of 300 randomly selected registered voters shows that the incumbent mayor of New Castle is leading her opponent by a 52% to 48% margin. Find the margin of error for the poll assuming a 95% confidence level.

Margin of Error (95% confidence level) = ± \( \frac{0.98 \sqrt{300}}{300} \)

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The margin of error assuming a 95% confidence level is 0.057, or ±5.7%. This means the mayor’s actual support is 95% likely to be between 46.3% and 57.7%.

Exercises

1. Weather

A meteorologist places ten weather stations in a county to measure rainfall. The median annual rainfall is calculated for the sample.

Sample: the 10 weather stations

Population: all locations in the county

Sample statistic: median annual rainfall in the sample

Population parameter: median annual rainfall across the county

2. Botany

A scientist randomly selects 20 trees in a forest. The mean height of the 20 trees is then calculated.

Sample: the 20 selected trees

Population: all trees in the forest

Sample statistic: mean height of trees in the sample

Population parameter: mean height of trees in the forest

3. Politics

A political reporter randomly selects 25 congressional districts across the country. The mean number of votes cast in the 25 congressional districts is calculated.

Sample: the 25 selected congressional districts

Population: all congressional districts nationwide

Sample statistic: mean number of votes cast in the sample

Population parameter: mean number of votes cast nationwide
### 12-3 Study Guide and Intervention (continued)  
**Statistics and Parameters**

**Statistical Analysis** The mean absolute deviation is the average of the absolute values of the differences between the mean and each value in the data set. It is used to predict errors and judge equality. The standard deviation is the calculated value that shows how data deviate from the mean of the set of data. The variance of data is the square of the standard deviation.

#### Example

**EMPLOYMENT** Employees at a law firm keep track of how many hours they work each week: {44, 48, 44, 40, 59}.

**a.** Find the mean absolute deviation.

**Step 1** Find the mean. For this set of data, the mean is 47.

**Step 2** Find the sum of the absolute values of the differences between each value in the set of data and the mean. 

$$|44 - 47| + |48 - 47| + |44 - 47| + |40 - 47| + |59 - 47| = 3 + 1 + 3 + 7 + 12 = 26$$

**Step 3** Divide the sum by the number of values in the set of data:

$$\frac{26}{5} \neq 5.2$$

The mean absolute deviation is 5.2.

**b.** Find the variance and standard deviation.

**Step 1** To find the variance, square the difference between each number and the mean. Then divide by the number of values.

$$\sigma^2 = \frac{(44 - 47)^2 + (48 - 47)^2 + (44 - 47)^2 + (40 - 47)^2 + (59 - 47)^2}{5}$$

$$\sigma^2 = \frac{(-3)^2 + (-1)^2 + (-3)^2 + (-7)^2 + (12)^2}{5} = \frac{9 + 1 + 9 + 49 + 144}{5} = \frac{188}{5}$$

**Step 2** The standard deviation is the square root of the variance.

$$\sigma = \sqrt{\frac{188}{5}} \approx 6.13$$

The variance of the data set is $\frac{188}{5}$ and the standard deviation is approximately 6.13.

### Exercises

Find the mean, mean absolute deviation, variance, and standard deviation for each set of data.

1. {2, 4, 9, 5}  
   $$\bar{x} = 5; \text{mean absolute deviation} = 2; \quad \bar{x} = 17; \text{mean absolute deviation} = 2; \quad \sigma^2 = 5.2; \quad \sigma = 2.28$$

2. {13, 17, 17, 22, 16}  
   $$\bar{x} = 17; \text{mean absolute deviation} = 2; \quad \bar{x} = 17; \text{mean absolute deviation} = 2; \quad \sigma^2 = 8.4; \quad \sigma = 2.90$$

### 12-3 Skills Practice  
**Statistics and Parameters**

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

1. **RESTAURANTS** A restaurant randomly selects 10 patrons on Saturday night. The median amount spent on beverages is then calculated for the sample.
   - **Sample:** the 10 selected patrons
   - **Population:** all patrons on Saturday night
   - **Sample statistic:** median amount spent on beverages in the sample
   - **Population parameter:** median amount spent on beverages on Saturday night

2. **KITTENS** A veterinarian randomly selects 3 kittens from a litter. The mean weight of the 3 kittens is calculated.
   - **Sample:** the 3 selected kittens
   - **Population:** all kittens in the litter
   - **Sample statistic:** the mean weight of the kittens in the sample
   - **Population parameter:** the mean weight of the kittens in the litter

3. **PRODUCE** A produce clerk randomly selects 20 bags of apples from each week's shipment and counts the total number of apples in each bag. The mode number of apples is calculated for the sample.
   - **Sample:** the 20 selected bags of apples
   - **Population:** all bags of apples in the week's shipment
   - **Sample statistic:** mode number of apples in the sample
   - **Population parameter:** mode number of apples in the week's shipment

4. **WILDLIFE** A researcher counts the number of river otters observed on each acre of land in a state park: {10, 14, 6, 0, 8, 4}.

5. **FISHING** A fisherman records the weight of each black bass he catches during a fishing trip: {12, 7, 8, 13, 6, 14}.

6. **BUDGETING** Xavier keeps track of how much money he spends on gasoline each week: {20, 13, 26, 0, 33, 16, 18}.
   - **Sample statistic:** median amount spent on gasoline in the sample
   - **Population parameter:** median amount spent on gasoline in each week

Find the mean, variance, and standard deviation of each set of data.

7. {2, 0, 10, 4}  
   $$\bar{x} = 4; \quad \sigma^2 = 14; \quad \sigma = 3.74$$
   $$\bar{x} = 7; \quad \sigma^2 = 1.5; \quad \sigma = 1.22$$

8. {6, 7, 6, 9}  
   $$\bar{x} = 7; \quad \sigma^2 = 1.5; \quad \sigma = 1.22$$
   $$\bar{x} = 5; \quad \sigma^2 = 8; \quad \sigma = 2.83$$

9. {10, 9, 13, 6, 7}  
   $$\bar{x} = 9; \quad \sigma^2 = 6; \quad \sigma = 2.45$$
   $$\bar{x} = 5; \quad \sigma^2 = 8; \quad \sigma = 2.83$$

10. {10, 6, 8, 2, 3, 2, 9}  
    $$\bar{x} = 6; \quad \sigma^2 = 2.25; \quad \sigma = 1.5$$

11. {20, 18, 25, 35}  
    $$\bar{x} = 25; \quad \sigma^2 = 2.25; \quad \sigma = 1.5$$

12. {44, 35, 50, 37, 43, 38, 40}  
    $$\bar{x} = 38; \quad \sigma^2 = 2.25; \quad \sigma = 1.5$$

13. **PARKING** A city councilor wants to know how much revenue the city would earn by installing parking meters on Main Street. He counts the number of cars parked on Main Street each weekday: {64, 79, 81, 53, 63}.
   - **Find the standard deviation.** 10.55
### 12-3 Practice

**Statistics and Parameters**

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

1. **MARINE BIOLOGY** A marine biologist randomly selects 30 oysters from a research tank. The mean weight of the 30 oysters is calculated.
   - **Sample:** the 30 selected oysters
   - **Population:** all oysters in the research tank
   - **Sample statistic:** mean weight of oysters in the sample
   - **Population parameter:** mean weight of oysters in the research tank

2. **CIVIL ENGINEERING** A civil engineer randomly selects 5 city intersections with traffic lights. The median length of a red light is calculated for the sample.
   - **Sample:** the 5 city intersections with traffic lights
   - **Population:** all city intersections with traffic lights
   - **Sample statistic:** median length of a red light in the sample
   - **Population parameter:** median length of a red light at all city intersections

3. **BASEBALL** A baseball commissioner randomly selects 10 home games played by a major league team. The median attendance is calculated for the games in the sample.
   - **Sample:** the 10 selected home games played by the team
   - **Population:** all home games played by the team
   - **Sample statistic:** median attendance for the games in the sample
   - **Population parameter:** median attendance for all home games played by the team

Find the mean absolute deviation.

4. **INVESTING** A stock broker keeps a record of the daily closing price of a share of stock in Bicom Corporation: 165.20, 46.10, 46.85, 42.55, 40.80.
   - **Sample:** the 5 selected prices
   - **Population:** all prices of the stock
   - **Sample statistic:** mean price
   - **Population parameter:** mean price

5. **GOLF** A golfer keeps track of his scores for each round: 78, 81, 86, 77, 75.
   - **Sample:** the 5 selected scores
   - **Population:** all scores of the player
   - **Sample statistic:** mean score
   - **Population parameter:** mean score

6. **WEATHER** A meteorologist keeps track of the number of thunderstorms occurring each month in Sussex County: 20, 4, 7, 1, 3, 5, 2.
   - **Sample:** the 7 selected months
   - **Population:** all months in Sussex County
   - **Sample statistic:** mean number of thunderstorms
   - **Population parameter:** mean number of thunderstorms

Find the mean, variance, and standard deviation of each set of data.

7. 6, 11, 16, 9
   - **x̄ = 10.5, σ² = 53 / 4, σ = 3.64**

8. 2, 5, 8, 11, 4
   - **x̄ = 6, σ² = 10, σ = 3.16**

9. 23.4, 16.8, 9.7, 22.1
   - **x̄ = 18, σ² = 29.075, σ = 5.39**

10. [2, 5, 1, 2, 3]
    - **x̄ = 11 / 4, σ² = 139 / 48, σ = 1.70**

11. 14, 166, 171, 150, 88
    - **x̄ = 144, σ² = 877.2, σ = 29.62**

12. [13, 24, 22, 17, 14, 29, 15, 22]
    - **x̄ = 19.5, σ² = 27.75, σ = 5.27**

13. **QUALITY CONTROL** An inspector checks each automobile that comes off of the assembly line. He keeps a record of the number of defective cars each day: 3, 1, 2, 0, 0, 4, 3, 6, 1, 21. Find the standard deviation.
    - **σ ≈ 1.78**

### 12-3 Word Problem Practice

**Statistics and Parameters**

1. **GAS PRICES** Renee is planning a road trip to her aunt's house. To estimate how much the trip will cost, she goes online and finds the price of a gallon of gasoline for 5 randomly selected gas stations along the route. She then calculates the mean price per gallon for the 5 selected gas stations.
   - **Sample:** the 5 selected gas stations
   - **Population:** all gas stations along the route
   - **Sample statistic:** mean price per gallon
   - **Population parameter:** mean price per gallon

2. **CIVIL ENGINEERING** A civic engineer randomly selects 5 city intersections with traffic lights. The median length of a red light is calculated for the sample.
   - **Sample:** the 5 city intersections with traffic lights
   - **Population:** all city intersections with traffic lights
   - **Sample statistic:** median length of a red light
   - **Population parameter:** median length of a red light

3. **BASEBALL** A baseball commissioner randomly selects 10 home games played by a major league team. The median attendance is calculated for the games in the sample.
   - **Sample:** the 10 selected home games played by the team
   - **Population:** all home games played by the team
   - **Sample statistic:** median attendance for the games in the sample
   - **Population parameter:** median attendance for all home games played by the team

4. **ERROR ANALYSIS** Myau and Alice are studying for their test on statistics. Renee is planning a road trip to her aunt's house. To estimate how much the trip will cost, she goes online and finds the price of a gallon of gasoline for 5 randomly selected gas stations along the route. She then calculates the mean price per gallon for the 5 selected gas stations.
   - **Sample:** the 5 selected gas stations
   - **Population:** all gas stations along the route
   - **Sample statistic:** mean price per gallon
   - **Population parameter:** mean price per gallon

   **Part a.** Identify the sample and the population.
   - **Sample:** the 5 selected gas stations
   - **Population:** all gas stations along the route
   - **Sample statistic:** mean price per gallon
   - **Population parameter:** mean price per gallon

   **Part b.** Identify the sample and the population.
   - **Sample:** the 5 selected gas stations
   - **Population:** all gas stations along the route
   - **Sample statistic:** mean price per gallon
   - **Population parameter:** mean price per gallon

   **Part c.** Describe the sample statistic and the population parameter.
   - **Sample statistic:** mean price per gallon
   - **Population parameter:** mean price per gallon

   **Part d.** Describe the sample statistic and the population parameter.
   - **Sample statistic:** mean price per gallon
   - **Population parameter:** mean price per gallon

5. **HEALTH CLUBS** To plan their future equipment purchases, the Northville Health Club randomly chooses 8 patrons and tracks how many minutes they spend on the treadmill: 30, 30, 45, 20, 60, 30, 30, 15. The mean treadmill time is then calculated.
   - **Sample:** the 8 selected patrons
   - **Population:** all patrons using the treadmill
   - **Sample statistic:** mean time on the treadmill
   - **Population parameter:** mean time on the treadmill

6. **BASEBALL** A baseball commissioner randomly selects 10 home games played by a major league team. The median attendance is calculated for the games in the sample.
   - **Sample:** the 10 selected home games played by the team
   - **Population:** all home games played by the team
   - **Sample statistic:** median attendance for the games in the sample
   - **Population parameter:** median attendance for all home games played by the team

7. **GOLF** A golfer keeps track of his scores for each round: 78, 81, 86, 77, 75.
   - **Sample:** the 5 selected rounds
   - **Population:** all rounds of the golfer
   - **Sample statistic:** mean score
   - **Population parameter:** mean score

8. **WEATHER** A meteorologist keeps track of the number of thunderstorms occurring each month in Sussex County: 20, 4, 7, 1, 3, 5, 2.
   - **Sample:** the 7 selected months
   - **Population:** all months in Sussex County
   - **Sample statistic:** mean number of thunderstorms
   - **Population parameter:** mean number of thunderstorms

9. **INVESTING** A stock broker keeps a record of the daily closing price of a share of stock in Bicom Corporation: 165.20, 46.10, 46.85, 42.55, 40.80.
   - **Sample:** the 5 selected prices
   - **Population:** all prices of the stock
   - **Sample statistic:** mean price
   - **Population parameter:** mean price

10. **GOLF** A golfer keeps track of his scores for each round: 78, 81, 86, 77, 75.
    - **Sample:** the 5 selected rounds
    - **Population:** all rounds of the golfer
    - **Sample statistic:** mean score
    - **Population parameter:** mean score

11. **WEATHER** A meteorologist keeps track of the number of thunderstorms occurring each month in Sussex County: 20, 4, 7, 1, 3, 5, 2.
    - **Sample:** the 7 selected months
    - **Population:** all months in Sussex County
    - **Sample statistic:** mean number of thunderstorms
    - **Population parameter:** mean number of thunderstorms

12. **INVESTING** A stock broker keeps a record of the daily closing price of a share of stock in Bicom Corporation: 165.20, 46.10, 46.85, 42.55, 40.80.
    - **Sample:** the 5 selected prices
    - **Population:** all prices of the stock
    - **Sample statistic:** mean price
    - **Population parameter:** mean price

13. **GOLF** A golfer keeps track of his scores for each round: 78, 81, 86, 77, 75.
    - **Sample:** the 5 selected rounds
    - **Population:** all rounds of the golfer
    - **Sample statistic:** mean score
    - **Population parameter:** mean score
Chapter 12

**12-3 Enrichment**

**Standard Error and Confidence Limits**

You just learned about standard deviation, which measures how widely spread data points in a sample are—essentially, how far data points are from the mean.

Another key statistical measure is the standard error of the mean, or \( S_e \). The standard error of the mean describes the uncertainty of how the sample mean represents the population mean, and is given by the equation \( S_e = \frac{\sigma}{\sqrt{n}} \), where \( \sigma \) is the standard deviation and \( n \) is the sample size.

The equation for the standard error indicates that the larger the sample size is, the smaller the standard error will be. The standard error is often described in terms of percentages called confidence limits.

95% confidence limit interval = \( \bar{x} \pm 1.96S_e \)

99% confidence limit interval = \( \bar{x} \pm 2.58S_e \)

**Example**

The standard deviation of the weight of individual apples at Mr. Maguire’s stand is 68 ounces. The mean weight of a sample of 25 apples is 5.2 ounces. Find the 95% confidence interval for the mean.

First, calculate the standard error.

\[ S_e = \frac{\sigma}{\sqrt{n}} = \frac{0.8}{\sqrt{25}} = 0.16 \]

Then, use the equation above for the 99% confidence interval.

99% confidence limit interval = \( \bar{x} \pm 2.58S_e \)

= \( 5.2 \pm 2.58(0.16) = 5.2 \pm 0.4128 \)

The 99% confidence interval is 4.7872 to 5.6128. This means we can be 99% sure that the population mean is between 4.7872 and 5.6128.

**Exercises**

Find the standard error of the mean for each data set.

1. (806, 313, 301, 293, 326, 315)
   - \( \approx 4.31 \)  
   - \( \approx 0.53 \)

2. (0.50, 0.43, 0.27, 0.64, 0.62, 0.60)
   - \( \approx 0.53 \)

3. CHEESE The local supermarket packages fresh cheese in refrigerated cases for its customers’ convenience. The weights of the packages of parmesan currently in the case, in ounces, are: 114, 12, 16, 17, 12, 16, 14, 15.
   - a. Find the standard error for the mean of the data set. \( \approx 0.72 \)
   - b. Find the 95% confidence interval for the mean. \( \approx 16.59 \) to \( 16.41 \)

4. CHILD CARE The standard deviation of the ages of the children at the Happy Smiles Child Care Center is 1.2 years. The mean age of a sample of 9 children was 7.4 years.
   - a. Find the 95% confidence interval for the mean. \( 6.616 \) to \( 8.184 \)
   - b. How many children must be included in the sample to be 95% certain that the estimate of the mean has an error of plus or minus 0.5 years? \( \geq 23 \) children

Chapter 12

**12-4 Study Guide and Intervention**

**Permutations and Combinations**

**Permutations** An arrangement or listing in which order or placement is important is called a permutation. For example, the arrangement AB of choices A and B is different from the arrangement BA of these same two choices.

**Example 1** Find \( P(6, 2) \).

\[ P(n, r) = \frac{n!}{(n-r)!} \]

\[ P(6, 2) = \frac{6!}{(6-2)!} = \frac{720}{24} = 30 \]

There are 30 permutations of 6 objects taken 2 at a time.

**Example 2** PASSWORDS A specific program requires the user to enter a 5-digit password. The digits cannot repeat and can be any five of the digits 1, 2, 3, 4, 7, 8, and 9.

- a. How many different passwords are possible?
  - Number of favorable outcomes = 5 · 4 · 3 · 2 · 1 = 120
  - Number of possible outcomes = 7 · 6 · 5 · 4 · 3 = 2520
  - There are 2520 ways to create a password.

- b. What is the probability that the first two digits are odd numbers with the other digits any of the remaining numbers?
  - Number of favorable outcomes = 4 · 3 · 2 · 1 = 24
  - Number of possible outcomes = 7 · 6 · 5 · 4 · 3 = 2520
  - The probability is \( \frac{24}{2520} = 0.00952 \) or 0.952%.

**Exercises**

Evaluate each expression.

1. \( P(7, 4) \)
2. \( P(12, 7) \)
3. \( P(9, 9) \)

4. **CLUBS** A club with ten members wants to choose a president, vice-president, secretary, and treasurer. Six of the members are women, and four are men.
   - a. How many different sets of officers are possible? 5040
   - b. What is the probability that all officers will be women. 7.1%
Combination

A club with ten members wants to choose a committee of four members. Six of the members are women, and four are men.

a. How many different committees are possible?

\[ C(n, r) = \frac{n!}{(n-r)! r!} \]

\[ n = 10, \quad r = 4 \]

\[ C(10, 4) = \frac{10!}{(10-4)! 4!} \]

\[ = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4!} \]

\[ = 210 \]

There are 210 ways to choose a committee of four when order is not important.

b. If the committee is chosen randomly, what is the probability that two members of the committee are men?

There are \( C(4, 2) = \frac{4!}{(4-2)! 2!} = 6 \) ways to choose two men randomly, and there are \( C(6, 2) = \frac{6!}{(6-2)! 2!} = 15 \) ways to choose two women randomly. By the Fundamental Counting Principle, there are 6 \cdot 15 or 90 ways to choose a committee with two men and two women.

Probability (2 men and 2 women) = \( \frac{number \ of \ favorable \ outcomes}{number \ of \ possible \ outcomes} \)

\[ = \frac{90}{210} \ or \ about \ 42.9\% \]

Exercises

Evaluate each expression.

1. \( C(7, 3) \)
2. \( C(12, 8) \)
3. \( C(9, 2) \)
4. \( P(6, 3) \)
5. \( P(7, 3) \)

6. \( P(9, 4) \)
7. \( P(7, 5) \)
8. \( P(5, 3) \)

9. \( C(6, 2) \)
10. \( C(9, 7) \)
11. \( C(8, 4) \)

12. \( C(7, 5) \)
13. \( C(12, 2) \)
14. \( C(13, 7) \)

15. \( C(11, 2) \)
16. \( P(5, 3) \)
17. \( C(14, 5) \)

18. \( C(11, 6) \)
19. \( P(4, 2) \)
20. \( C(8, 6) \)

Evaluate each expression.

1. \( C(7, 3) = 35 \)
2. \( C(12, 8) = 495 \)
3. \( C(9, 2) = 36 \)

4. COMMITTEES In how many ways can a club with 9 members choose a two-member sub-committee? 36

5. BOOK CLUBS A book club offers its members a book each month for a year from a selection of 24 books. Ten of the books are biographies and 14 of the books are fiction.

a. How many ways could the members select 12 books? 2,704,156
b. What is the probability that 5 biographies and 7 fiction books will be chosen? about 32%
**Practice**

Permutations and Combinations

Use the Fundamental Counting Principle to evaluate each of the following.

1. **ERRANDS** Wesley needs to stop at 6 stores on the way home from work. How many ways can Wesley arrange the 6 stops he needs to make? 720

2. **VOTING** There are 8 people waiting in line to cast their votes. How many ways can the people line up to vote? 40,320

Evaluate each expression.

3. \( P(11, 3) = 990 \)

4. \( P(6, 3) = 120 \)

5. \( P(15, 3) = 2730 \)

6. \( C(10, 9) = 10 \)

7. \( C(12, 9) = 220 \)

8. \( C(7, 3) = 35 \)

9. \( C(7, 4) = 35 \)

10. \( C(12, 4) = 495 \)

11. \( P(13, 3) = 1716 \)

12. \( C(16, 12) = 1820 \)

13. \( C(17, 2) = 136 \)

14. \( C(16, 15) = 16 \)

15. \( P(20, 5) = 1,860,480 \)

16. \( P(11, 7) = 1,663,200 \)

17. \( P(13, 1) = 13 \)

18. \( C(19, 16) = 969 \)

19. \( P(15, 4) = 32,760 \)

20. \( C(14, 7) = 3432 \)

19. \( P(15, 4) = 32,760 \)

21. **SPORTS** In how many orders can the top five finishers in a race finish? 120

22. **JUDICIAL PROCEDURE** The court system in a community needs to assign 3 out of 8 judges to a docket of criminal cases. Five of the judges are male and three are female.

a. Does the selection of judges involve a permutation or a combination? **Combination**

b. In how many ways could three judges be chosen? 56

c. If the judges are chosen randomly, what is the probability that all 3 judges are male? 5/26, or about 18%

**Word Problem Practice**

Permutations and Combinations

1. **CHORES** In Ashley’s family there are 4 children and their mother and father. If two of the six must help clear the table after dinner, how many ways can two people be paired? 15

2. **PUBLIC SERVICE** A county computer randomly selects jurors from the lawyer-approved potential juror list. How many ways are there for 12 jurors to be chosen from a pool of 20? 125,970

3. **SPORTS** Hans’ basketball team decides to choose two captains each week so that many players get the chance to be captain. Each week, each of the 11 players writes his name on a slip of paper. The papers are then placed in a container and mixed. The last week’s captains draw two slips of paper from the container; these two people are captains for the following week. How many different pairs of captains can be formed? 55

4. **PUZZLES** A popular newspaper puzzle involves a series of letters that can be rearranged to form a word. Will is writing his own version of the game for a school project. He wants to include the following word for his puzzle.

   **numbers**

   How many ways are there to arrange the letters with the letter b as the first letter? 720

5. **HORSE RACING** In the 133rd running of the Kentucky Derby in 2007, there were 22 contenders.

   a. How many different ways can the horses finish the race? 22!

   b. How many different ways can horses place first, second, and third? 6840

   c. If all 22 horses have an equal chance of winning and 3 of the horses are female, what is the probability that a female horse places first, second, and third? 6/22
A Great Pizza Deal

A television commercial advertised a pizza deal in which customers could choose two pizzas each with up to five toppings chosen from a set of eleven toppings. On the commercial, a boy claims that there are $1024^2$ or $1,048,576$ ways for the customer to choose the two pizzas. Is this a valid claim?

For a limited time, you can choose 2 pizzas with up to 5 toppings for only $7.99!!!!!!

1. Suppose the customer can have as many toppings, up to eleven, as desired. Using counting techniques, how many different ways can the pizzas be made? (Hint: Imagine that the customer has to choose “yes” or “no” for each topping.) $2^{11}$ or 2048

2. Use combinations to determine how many ways the customer can make one pizza with up to five toppings. $C(11, 0) + C(11, 1) + C(11, 2) + C(11, 3) + C(11, 4) + C(11, 5)$ or 1024

3. How many ways can the two pizzas be built if they are identical? 1024

4. How many ways can the two pizzas be built if they are different? (Hint: Use the total number of ways that one pizza can be made from problem 2 and then choose 2 of those possibilities.) Should you use a combination or a permutation? combination; $C(1024, 2)$ or 523,776

5. Add the results from problems 3 and 4 to determine the total number of ways to make two pizzas with up to five toppings each. 524,800

6. The boy in the ad claims that the customer has $1024^2$ choices. What is $1024^2$? How does this compare to the claim made by the boy in the advertisement? 1,048,576; You actually have a lot fewer choices than the boy claims.

### Example 1
Find the probability that you will roll a six and then a five when you roll a die twice.

By the definition of independent events, $P(A \text{ and } B) = P(A) \cdot P(B)$

First roll: $P(6) = \frac{1}{6}$

Second roll: $P(5) = \frac{1}{6}$

$P(6 \text{ and } 5) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

The probability that you will roll a six and then a five is $\frac{1}{36}$.

### Example 2
A bag contains 3 red marbles, 2 green marbles, and 4 blue marbles. Two marbles are drawn randomly from the bag and not replaced. Find the probability that both marbles are blue.

By the definition of dependent events, $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

First marble: $P(\text{blue}) = \frac{4}{9}$

Second marble: $P(\text{blue, blue}) = \frac{4}{9} \cdot \frac{3}{8} = \frac{12}{72}$

$P(\text{blue, blue}) = \frac{1}{6}$

The probability of drawing two blue marbles is $\frac{1}{6}$.

### Exercises
A bag contains 3 red, 4 blue, and 6 yellow marbles. One marble is selected at a time, and once a marble is selected, it is not replaced. Find each probability.

1. $P(2 \text{ yellow}) = \frac{3}{26}$
2. $P(\text{red, yellow}) = \frac{3}{26}$
3. $P(\text{blue, red, yellow}) = \frac{5}{143}$

4. George has two red socks and two white socks in a drawer. What is the probability of picking a red sock and a white sock in that order if the first sock is not replaced? $\frac{1}{3}$

5. Phyllis drops a penny in a pond, and then she drops a nickel in the pond. What is the probability that both coins land with tails showing? $\frac{1}{4}$

6. A die is rolled and a penny is dropped. Find the probability of rolling a two and showing a tail. $\frac{1}{12}$
**Probability of Compound Events**

### Mutually Exclusive and Inclusive Events

Events that cannot occur at the same time are called mutually exclusive. If two events are not mutually exclusive, they are called inclusive.

**Example**

A card is drawn from a standard deck of playing cards. Find the probability of drawing a king or a queen.

Drawing a king or a queen are mutually exclusive events. By the definition of mutually exclusive events, $P(A \text{ or } B) = P(A) + P(B)$.

- $P(\text{king}) = \frac{4}{52} = \frac{1}{13}$
- $P(\text{queen}) = \frac{4}{52} = \frac{1}{13}$

Thus, $P(\text{king or queen}) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}

The probability of drawing a king or a queen is $\frac{2}{13}$.

### Exercises

A bag contains 2 red, 5 blue, and 7 yellow marbles. Find each probability.

1. $P(\text{yellow or red}) = \frac{9}{14}$
2. $P(\text{red or not yellow}) = \frac{1}{2}$
3. $P(\text{blue or red or yellow}) = 1$

A card is drawn from a standard deck of playing cards. Find each probability.

4. $P(\text{jack or red}) = \frac{7}{13}$
5. $P(\text{red or black}) = 1$
6. $P(\text{jack or clubs}) = \frac{4}{13}$
7. $P(\text{queen or less than 3}) = \frac{13}{13}$
8. $P(5 \text{ or } 6) = \frac{2}{13}$

10. In a math class, 12 out of 15 girls are 14 years old and 14 out of 17 boys are 14 years old. What is the probability of selecting a girl or a 14-year old from this class?

A die is rolled and a spinner like the one at the right is spun. Find each probability.

7. $P(4 \text{ and } A) = \frac{4}{13}$

8. $P(\text{an even number and } C) = \frac{3}{4}$

9. $P(2 \text{ or } 5 \text{ and } B \text{ or } D) = \frac{1}{6}$

10. $P(\text{a number less than 5 and } B, C, \text{ or } D) = \frac{1}{2}$

A die is rolled and a spinner like the one at the right is spun. Find each probability.

11. $P(\text{jack or ten}) = \frac{2}{13}$
12. $P(\text{red or black}) = 1$
13. $P(\text{queen or clubs}) = \frac{4}{13}$
14. $P(\text{red or ace}) = \frac{7}{13}$
15. $P(\text{diamond or black}) = \frac{3}{4}$
16. $P(\text{face card or spade}) = \frac{11}{26}$

Tiles numbered 1 through 20 are placed in a box. Tiles numbered 11 through 30 are placed in a second box. The first tile is randomly drawn from the first box. The second tile is randomly drawn from the second box. Find each probability.

17. $P(\text{both are greater than } 15) = \frac{3}{16}$
18. The first tile is odd and the second tile is less than 25.
19. The first tile is a multiple of 6 and the second tile is a multiple of 4.
20. The first tile is less than 15 and the second tile is even or greater than 25.
A bag contains 5 red, 3 brown, 6 yellow, and 2 blue marbles. Once a marble is selected, it is not replaced. Find each probability.

1. $P(\text{brown, then yellow, then red}) = \frac{3}{112}$
2. $P(\text{red, then red, then blue}) = \frac{1}{84}$
3. $P(\text{yellow, then yellow, then not blue}) = \frac{3}{28}$
4. $P(\text{brown, then brown, then not yellow}) = \frac{1}{70}$

A die is rolled and a card is drawn from a standard deck of 52 cards. Find each probability.

5. $P(6 \text{ and king}) = \frac{1}{156}$
6. $P(\text{odd number and black}) = \frac{1}{4}$
7. $P(\text{less than 3 and heart}) = \frac{1}{12}$
8. $P(\text{greater than 1 and black ace}) = \frac{5}{156}$

9. A card is being drawn from a standard deck of playing cards. Determine whether the events are mutually exclusive or not mutually exclusive. Then find the probability.
   a. $P(\text{spade or numbered card}) = \frac{43}{52}$
   b. $P(\text{ace or red queen}) = \frac{3}{26}$
   c. $P(\text{heart or not face card}) = \frac{49}{52}$

Tiles numbered 1 through 25 are placed in a box. Tiles numbered 11 through 30 are placed in a second box. The first tile is randomly drawn from the first box. The second tile is randomly drawn from the second box. Find each probability.

10. The first tile is greater than 10 and the second tile is less than 25 or even.
11. The first tile is a multiple of 3 or prime and the second tile is a multiple of 5.
12. The first tile is less than 9 or odd and the second tile is a multiple of 4 or less than 21.

At Corrugated Packaging, Inc., a team of six employees is in charge of marketing and selling the company’s products; three are women and three are men. The president of the company decides to send four team members to a national cardboard box conference. He wants to make sure he chooses names fairly, so he decides to put the names of his six sales employees in a hat and draw four names to see who will go to the conference. What is the probability that the team will consist of three women and one man?

A die is rolled and a card is drawn from a standard deck of 52 cards. Find each probability.

13. The first tile is greater than 10 and the second tile is less than 25 or even.
14. The first tile is a multiple of 3 or prime and the second tile is a multiple of 5.
15. The first tile is less than 9 or odd and the second tile is a multiple of 4 or less than 21.

The forecast predicts a 40% chance of rain on Tuesday and a 60% chance on Wednesday. If these probabilities are independent, what is the chance that it will rain on both days?

Tomaso places favorite recipes in a bag for 4 pasta dishes, 5 casseroles, 3 types of chili, and 8 desserts.

- a. If Tomaso chooses one recipe at random, what is the probability that he selects a pasta dish or a casserole?
- b. If Tomaso chooses one recipe at random, what is the probability that he does not select a dessert?
- c. If Tomaso chooses two recipes at random without replacement, what is the probability that the first recipe he selects is a casserole and the second recipe he selects is a dessert?
Happy Birthday
On a reality show, a contestant must randomly choose enough people to go into a room so that the probability that at least one of those people has the same birthday (month and date) as the contestant is greater than 50%. Time is of the essence, so the contestant doesn’t want any more people than necessary. What number of people should the contestant gather?

1. Make a conjecture about the number of people needed. Answers will vary.

2. When the probability that every person in the room has a different birthday is less than 50%, the probability that two of them will share a birthday is greater than 50%. Explain why. Sample Answer: Because if the probability is less than 50%, then the complement will be 1 – which will be greater than 50%.

3. Observe this pattern:
   - If there are two people in the room, the probability that they all have different birthdays is \( \frac{365}{365} \cdot \frac{364}{365} \) or 99.7%.
   - If there are three people in the room, the probability that they all have different birthdays is \( \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \) or 99.2%.
   - If there are four people in the room, the probability that they all have different birthdays is \( \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \) or 99.2%.

   Extend this pattern until you get a probability less than 50%. How many people do you need to get a probability less than 50%? 24 people

4. So, how many people do you need to have in a room so that the probability of two of them sharing a birthday is greater than 50%? How does this compare to your answer to problem 1? 24 people; see students’ work.

5. Use the method above to determine how many people would need to be in a room so that the probability of two of them having birthdays on the same day of the month is greater than 50%. 7 people

---

**Enrichment**

**Happy Birthday**

On a reality show, a contestant must randomly choose enough people to go into a room so that the probability that at least one of those people has the same birthday (month and date) as the contestant is greater than 50%. Time is of the essence, so the contestant doesn’t want any more people than necessary. What number of people should the contestant gather?

1. Make a conjecture about the number of people needed. Answers will vary.

2. When the probability that every person in the room has a different birthday is less than 50%, the probability that two of them will share a birthday is greater than 50%. Explain why. Sample Answer: Because if the probability is less than 50%, then the complement will be 1 – which will be greater than 50%.

3. Observe this pattern:
   - If there are two people in the room, the probability that they all have different birthdays is \( \frac{365}{365} \cdot \frac{364}{365} \) or 99.7%.
   - If there are three people in the room, the probability that they all have different birthdays is \( \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \) or 99.2%.
   - If there are four people in the room, the probability that they all have different birthdays is \( \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \) or 99.2%.

   Extend this pattern until you get a probability less than 50%. How many people do you need to get a probability less than 50%? 24 people

4. So, how many people do you need to have in a room so that the probability of two of them sharing a birthday is greater than 50%? How does this compare to your answer to problem 1? 24 people; see students’ work.

5. Use the method above to determine how many people would need to be in a room so that the probability of two of them having birthdays on the same day of the month is greater than 50%. 7 people

---

**TI-Nspire Activity**

**Probability of Compound Events**

The TI-Nspire calculator can be used to evaluate permutations and combinations using the functions \( nPr \) and \( nCr \), respectively.

**Example 1** Evaluate each expression.

a. \( P(8, 3) \)

Calculate the number of permutations of eight objects taken three at a time. Keystrokes: \( nP \) \( r \) \( (8, 3) \)

b. \( C(8, 3) \)

Calculate the number of combinations of eight objects taken three at a time. Keystrokes: \( nC \) \( r \) \( (8, 3) \)

---

**Example 2** What is the probability of drawing five cards from a standard deck of cards and having three of the cards be aces? Write the probability as the product of combinations for having three aces over the total combinations for any five cards:

\[ \frac{C(13, 1) \cdot C(4, 3) \cdot C(48, 2)}{C(52, 5)} \]

Calculate the probability using the \( nCr \) command.

Keystrokes: \( nC \) \( r \) \( (52, 5) \)

The probability is \( \frac{94}{4165} \)

---

**Exercises**

Evaluate each expression.

1. \( P(8, 5) \)

2. \( C(12, 8) \)

3. \( P(7, 3) \)

4. \( C(52, 5) \)

5. \( P(9, 4) \)

6. \( C(48, 4) \)

7. \( P(13, 4) \)

8. \( C(36, 5) \)

---

**Find each probability.**

9. What is the probability of drawing four 2s when selecting five cards from a standard deck of cards?

\[ \frac{48}{C(52, 5)} = \frac{48}{2,598,960} \]

10. What is the probability of rolling five dice and all of them showing the same number?
Chapter 12

NAME _______________________ DATE _______________ PERIOD ____________

12-6 Study Guide and Intervention

Probability Distributions

Random Variables and Probability

A random variable $X$ is a variable whose value is the numerical outcome of a random event.

Example

A teacher asked her students how many siblings they have. The results are shown in the table at the right. 

<table>
<thead>
<tr>
<th>Number of Siblings</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

a. Find the probability that a randomly selected student has 2 siblings.

The random variable $X$ can equal 0, 1, 2, 3, or 4. In the table, the value $X = 2$ is paired with 8 outcomes, and there are 27 students surveyed.

$P(X = 2) = \frac{2 \text{ siblings}}{27 \text{ students surveyed}} = \frac{8}{27}$

The probability that a randomly selected student has 2 siblings is $\frac{8}{27}$, or 29.6%.

b. Find the probability that a randomly selected student has at least three siblings.

$P(X \geq 3) = \frac{3 + 1}{27}$

The probability that a randomly selected student has at least 3 siblings is $\frac{4}{27}$, or 11.1%.

Exercises

For Exercises 1–3, use the grade distribution shown at the right. A grade of A = 5, B = 4, C = 3, D = 2, F = 1.

<table>
<thead>
<tr>
<th>$X$ = Grade</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>6</td>
<td>9</td>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

1. Find the probability that a randomly selected student in this class received a grade of C.

2. Find the probability that a randomly selected student in this class received a grade lower than a C.

3. What is the probability that a randomly selected student in this class passes the course, that is, gets at least a D?

4. The table shows the results of tossing 3 coins 50 times. What is the probability of getting 2 or 3 heads? 48%

Exercises

The table at the right shows the probability distribution for students by school enrollment in the United States in 2000. Use the table for Exercises 1–3.

<table>
<thead>
<tr>
<th>$X$ = Type of School</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary = 1</td>
<td>0.562</td>
</tr>
<tr>
<td>Secondary = 2</td>
<td>0.215</td>
</tr>
<tr>
<td>Higher Education = 3</td>
<td>0.223</td>
</tr>
</tbody>
</table>

1. Show that the distribution is valid.

For each value of $X$, the probability is greater than or equal to 0 and less than or equal to 1. Also, the sum of the probabilities is 1.

b. What is the probability that a student chosen at random has fewer than 2 siblings?

Because the events are independent, the probability of fewer than 2 siblings is the sum of the probability of 0 siblings and the probability of 1 sibling, or 0.037 + 0.046 = 0.083.

Exercises

1. Show that the distribution is valid.

$0 \leq P(X) \leq 1$; the sum of the probabilities = 1

2. If a student is chosen at random, what is the probability that the student is in elementary or secondary school? 0.777

3. Make a probability histogram of the data.
**12-6 Skills Practice**

**Probability Distributions**

For Exercises 1–3, the spinner shown is spun three times.

1. Write the sample space with all possible outcomes.
   
   GGG, GGB, GBG, BGG, BGB, BBG, BBB

2. Find the probability distribution $X$, where $X$ represents the number of times the spinner lands on green for $X = 0$, $X = 1$, $X = 2$, and $X = 3$.
   
   $P(X = 0) = 0.125$, $P(X = 1) = 0.375$, $P(X = 2) = 0.375$, $P(X = 3) = 0.125$

3. Make a probability histogram of the data.

For Exercises 4–6, the spinner shown is spun two times.

4. Write the sample space with all possible outcomes.
   
   RR, RB, RG, RV, BR, BB, BG, BY, GR, GB, GG, YR, YB, YG, YY

5. Find the probability distribution $X$, where $X$ represents the number of times the spinner lands on yellow for $X = 0$, $X = 1$, and $X = 2$.
   
   $P(X = 0) = 0.5625$, $P(X = 1) = 0.375$, $P(X = 2) = 0.0625$

6. Make a probability histogram of the data.

7. BUSINESS Use the table that shows the probability distribution of the number of minutes a customer spends at the express checkout at a supermarket.

<table>
<thead>
<tr>
<th>$X$ (Minutes)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.09</td>
<td>0.13</td>
<td>0.28</td>
<td>0.32</td>
<td>0.18</td>
</tr>
</tbody>
</table>

   a. Show that the distribution is valid. $0.09 + 0.13 + 0.28 + 0.32 + 0.18 = 1$
   
   b. What is the probability that a customer spends less than 3 minutes at the checkout? 0.22
   
   c. What is the probability that the customer spends at least 4 minutes at the checkout? 0.50

**12-6 Practice**

**Probability Distributions**

For Exercises 1–3, the spinner shown is spun two times.

1. Write the sample space with all possible outcomes.
   
   BB, BW, BR, BY, BG, WB, WW, WR, WY, RB, RW, RR, Ry, YB, YW, YA, YY, GB, GW, GR, YG

2. Find the probability distribution $X$, where $X$ represents the number of times the spinner lands on blue for $X = 0$, $X = 1$, and $X = 2$.
   
   $P(X = 0) = 0.64$, $P(X = 1) = 0.32$, $P(X = 2) = 0.04$

3. Make a probability histogram of the data.

4. TELECOMMUNICATIONS Use the table that shows the probability distribution of the number of telephones per student's household at Wilson High.

<table>
<thead>
<tr>
<th>$X$ (Number of Telephones)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.01</td>
<td>0.16</td>
<td>0.34</td>
<td>0.39</td>
<td>0.10</td>
</tr>
</tbody>
</table>

   a. Show that the distribution is valid. $0.01 + 0.16 + 0.34 + 0.39 + 0.10 = 1$
   
   b. If a student is chosen at random, what is the probability that there are more than 3 telephones at the student's home? 0.49
   
   c. Make a probability histogram of the data.

5. LANDSCAPING Use the table that shows the probability distribution of the number of shrubs (rounded to the nearest 50) ordered by corporate clients of a landscaping company over the past five years.

<table>
<thead>
<tr>
<th>$X$ (Number of Shrubs)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.11</td>
<td>0.24</td>
<td>0.45</td>
<td>0.16</td>
<td>0.04</td>
</tr>
</tbody>
</table>

   a. Define a random variable and list its values. Let $X$ = the number of shrubs ordered; $X = 50$, 100, 150, 200, 250
   
   b. Show that the distribution is valid. $0.11 + 0.24 + 0.45 + 0.16 + 0.04 = 1$
   
   c. What is the probability that a client's (rounded) order was at least 150 shrubs? 0.65
1. GAMES A spinner for an adventure game decides how a player will move. Write the sample space with all possible outcomes if the spinner is spun twice.

FF, FB, FL, FR, BF, BB, BL, BR, LF, LB, LL, LR, RF, RB, RL, RR

2. JOBS The probability distribution table shows the results of a newspaper survey that asked babysitters how much they make per hour.

Hourly Pay | Probability
---|---
$3 | 0.1
$4 | 0.2
$5 | 0.5
$6 | 0.15
$7 | 0.05

What is the probability that a babysitter chosen at random from the survey makes more than $5 per hour?

0.20 = 20%

3. TIME James looks at his 12-hour digital clock when he wakes up in the middle of the night. Make a probability histogram for all of the possible outcomes if you let X equal the first digit of the time. For instance, if it is 3:41, the first digit is a 3.

4. DARTS Suppose two darts are thrown at a traditional dart board and land near the center, but not on a bullseye. What is the probability that the sum of their scores is less than or equal to 5?

\[
\frac{10 \div 400}{0.025} = 2.5\% 
\]

5. TENNIS All season Oliver’s tennis coach has kept track of where each of his serves has landed during practice. The results are shown in the probability distribution table. A serve in region 2 is “in”, while a serve in any other region is a fault. A serve in region 1 represents hitting the net.

<table>
<thead>
<tr>
<th>Region</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.19</td>
</tr>
<tr>
<td>2</td>
<td>0.52</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
</tr>
</tbody>
</table>

a. What is the probability that any one of Oliver’s serves does not hit the net and is a fault?

0.29 or 29%

b. Make a probability histogram that represents the data in the table.

4. Binomial Distribution

A binomial distribution is a particular type of probability distribution. To determine a probability a binomial distribution uses the binomial coefficients and the formula

\[
\binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}
\]

where \(n\) is the number of selections, \(k\) is the desired outcome, and \(p\) is the probability of the desired outcome. So, if we have a bag that contains 4 red balls and 8 black balls, we can use this formula to determine the probability of choosing exactly 2 red balls out of 5 draws if we return the ball after each draw.

Use the formula to determine the probability:

\[
\binom{5}{2} \cdot \left(\frac{4}{12}\right)^2 \cdot \left(\frac{8}{12}\right)^3 = \frac{80}{243}
\]

So, the probability of choosing exactly 2 red balls out of 5 draws is \(\frac{80}{243}\).

Exercises

1. Determine the probability of drawing exactly 3 black balls.

2. Compare the probability found in problem 1 to the probability found in the example. Explain why they compare the way they do.

It is the same as the example because the probability of getting exactly 2 red balls is the same as the probability of getting exactly 3 black balls.

3. Determine the probability of drawing exactly 3 red balls.

4. Determine the probability of drawing exactly 4 red balls.

5. Now assume that there are 3 red balls and 2 black balls in the bag. Complete the probability distribution table using the binomial distribution.

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(0 Red)</td>
<td>(\frac{8}{125})</td>
</tr>
<tr>
<td>P(1 Red)</td>
<td>(\frac{36}{125})</td>
</tr>
<tr>
<td>P(2 Red)</td>
<td>(\frac{54}{125})</td>
</tr>
<tr>
<td>P(3 Red)</td>
<td>(\frac{27}{125})</td>
</tr>
</tbody>
</table>
### 12-6 Spreadsheet Activity
#### Probability Histograms

Spreadsheets have many formulas programmed including, combinations and permutations, that can be used to investigate probability.

**Example**

Use a spreadsheet to make a probability histogram for the number of heads when five coins are tossed.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter the numbers of heads, $X$, into Column A.</td>
</tr>
<tr>
<td>2</td>
<td>Use Column B to find the number of ways that $X$ heads can be achieved. If a coin is tossed 5 times, $X$ heads can be achieved in $C(5, X)$ ways.</td>
</tr>
<tr>
<td>3</td>
<td>Find the probabilities in Column C.</td>
</tr>
<tr>
<td>4</td>
<td>Select Column C to create the histogram. Choose Chart from the Insert menu. Then Choose the Column chart. Use the Series tab to designate the Category (X) axis labels as the data in cells A2 through A7. Use the menus to format the histogram.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X$</th>
<th>Number of Heads in 5 tosses</th>
<th>Ways to Achieve</th>
<th>$P(X)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>1</td>
<td>0.03125</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>5</td>
<td>0.15625</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>10</td>
<td>0.3125</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>10</td>
<td>0.3125</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5</td>
<td>0.15625</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0.03125</td>
</tr>
</tbody>
</table>

### Exercises

For Exercises 1–3, use the probability histogram for tossing 5 coins.

1. What is the probability that more than 3 heads are tossed? 0.1875 or 18.75%
2. What is the probability that fewer than 3 heads are tossed? 0.5 or 50%
3. Is this a valid probability distribution? Explain. Yes; each probability is greater than 0 and less than or equal to 1 and the sum of the probabilities is 1.
4. Use a spreadsheet to make a probability histogram for the number of heads when ten coins are tossed. See students' work.

---

### 12-7 Study Guide and Intervention
#### Probability Simulations

**Theoretical and Experimental Probability**

The probability used to describe events mathematically is called the theoretical probability. For example, the mathematical probability of rolling a 4 with a number cube is $\frac{1}{6}$, or $P(4) = \frac{1}{6}$. Experimental probability is the ratio of the number of times an outcome occurs in an experiment to the total number of events or trials, known as the relative frequency.

**Example 1**

Matt recorded that it rained 8 times in November and snowed 3 times. The other days, it was sunny. There are 30 days in November. Suppose Matt uses these results to predict November's weather next year. What is the probability that a day in November will be sunny?

\[
\text{Experimental Probability} = \frac{\text{frequency of outcome}}{\text{total number of trials}}
\]

\[
= \frac{19}{30} = 63.3\%
\]

The probability that it will be sunny on a day in November is 63.3%.

**Example 2**

A football team noticed that 9 of the last 20 coin tosses to choose which team would receive the ball first resulted in tails. What is the experimental probability of the coin landing on tails? What is the theoretical probability?

\[
\text{Experimental Probability} = \frac{\text{frequency of outcome}}{\text{total number of trials}}
\]

\[
= \frac{9}{20} = 45\%
\]

In this case, the experimental probability that a coin toss will be tails is 45%. If the coin is fair, the mathematical probability is 50%.

### Exercises

1. **DIE ROLL** A math class decided to test whether a die is fair, that is, whether the experimental probability equals the theoretical probability. The results for 100 rolls are shown at the right.

   | 1: 1 | 2: 15 |
---|------|-------|
| 3: 4 | 4: 10 |
| 5: 15| 6: 42 |

   a. What is the theoretical probability of rolling a 6? 16.7%
   b. What is the experimental probability of rolling a 6? 47%
   c. Is the die fair? Explain your reasoning. Probably not; theoretical probability ≠ experimental probability.
Probability Simulations

1. **CARDS**
   Use a standard deck of 52 cards. Select a card at random, record the suit of the card (heart, diamond, club, or spade), and then replace the card. Repeat this procedure 26 times.

   **a.** Based on your results, what is the experimental probability of selecting a heart?
   
   **Example**
   In one baseball season, Pete was able to get a base hit 42 of the 254 times he was at bat.

   **a.** What could be used to simulate his getting a base hit?
   
   **Answers**
   Spin the spinner once to simulate a time at bat. Let an outcome of 1 correspond to Pete’s getting a base hit. Let all other outcomes correspond to his not getting a hit.

   **b.** Describe a way to simulate his next 10 times at bat.
   
   **Answers**
   Let an outcome of 1 correspond to Pete’s getting a base hit. Let all other outcomes correspond to his not getting a hit. Spin the spinner once to simulate a time at bat. Record the result and repeat this 9 more times.

2. **SIBLINGS**
   There are 3 siblings in the Bencieva family. What could you use to simulate the genders of the 3 siblings?

   **Sample answer:** three coins, one for each of the siblings, with tails for male and heads for female, or 1 coin tossed three times.

3. **TRANSPORTATION**
   A random survey of 23 students revealed that 2 students walk to school, 12 ride the bus, 6 drive a car, and 3 ride with a parent or other adult. What could you use for a simulation to determine the probability that a student selected at random uses any one type of transportation?

   **Sample answer:** Use differently colored marbles that match in number with the survey for each transportation type.

4. **BIOLOGY**
   Stephen conducted a survey of the students in his classes to observe the distribution of eye color. The table shows the results of his survey.

<table>
<thead>
<tr>
<th>Eye Color</th>
<th>Blue</th>
<th>Brown</th>
<th>Green</th>
<th>Hazel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>12</td>
<td>5</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

   **a.** Find the experimental probability distribution for each eye color.
   
   \[P(Blue) = 0.15, \quad P(Brown) = 0.725, \quad P(Green) = 0.025, \quad P(Hazel) = 0.1\]

   **b.** Based on the survey, what is the experimental probability that a student in Stephen’s classes has blue or green eyes?
   
   **Answers**
   \[0.175\]

   **c.** Based on the survey, what is the experimental probability that a student in Stephen’s classes does not have green or hazel eyes?
   
   **Answers**
   \[0.875\]

   **d.** If the distribution of eye color in Stephen’s grade is similar to the distribution in his classes, about how many of the 360 students in his grade would you expect to have brown eyes?
   
   **Answers**
   About 261
1. **MARBLES** Place 5 red, 4 yellow, and 7 green marbles in a box. Randomly draw two marbles from the box, record each color, and then return the marbles to the box. Repeat this procedure 50 times.

   a. Based on your results, what is the experimental probability of selecting two yellow marbles? **Answers will vary. The theoretical probability is 0.05.**

   b. Based on your results, what is the experimental probability of selecting a green marble and a yellow marble? **Answers will vary. The theoretical probability is about 0.233.**

   c. Compare your results to the theoretical probabilities. The theoretical probability in Exercise 1 is 0.05, and in Exercise 2 is about 0.233.

2. **OPTOMETRY** Color blindness occurs in 4% of the male population. What could you use to simulate this situation? **Sample answer: a deck of playing cards in which 1 card is red and 24 are black.**

3. **SCHOOL CURRICULUM** Laurel Woods High randomly selected students for a survey to determine the most important school issues among the student body. The school wants to develop a curriculum that addresses these issues. The survey results are shown in the table.

<table>
<thead>
<tr>
<th>School Issues</th>
<th>Number Ranking</th>
<th>Issue Most Important</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grades</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>School Standards</td>
<td>17</td>
<td>17</td>
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<tr>
<td>Popularity</td>
<td>64</td>
<td>64</td>
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<tr>
<td>Dating</td>
<td>76</td>
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<tr>
<td>Violence</td>
<td>68</td>
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<tr>
<td>Drugs, including tobacco</td>
<td>29</td>
<td>29</td>
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   a. Find the experimental probability distribution of the importance of each issue. **P(Grades) = 0.119, P(School Standards) = 0.055, P(Popularity) = 0.270, P(Dating) = 0.244, P(Violence) = 0.219, P(Drugs) = 0.093.**

   b. Based on the survey, what is the experimental probability that a student chosen at random thinks the most important issue is grades or school standards? **about 0.174**

   c. The enrollment in the 9th and 10th grades at Laurel Woods High is 168. If their opinions are reflective of those of the school as a whole, how many of them would you expect to have chosen popularity as the most important issue? **about 45**

   d. Suppose the school develops a curriculum incorporating the top three issues. What is the probability that a student selected at random will think the curriculum addresses the most important issue at school? **about 0.733**

4. **GAMES** Suppose you spin the spinner below 20 times. You get 6 red, 4 blue, 5 yellow, and 5 green. What is the theoretical probability of spinning red? **25%; 30%**

**Word Problem Practice**

1. **EARTHQUAKES** Geologists conclude that there is a 62% probability of a magnitude 6.7 or greater quake striking the San Francisco Bay region before 2032. Does this represent empirical probability, theoretical probability, or experimental probability? **Empirical probability.**

2. **TOYS** There is a toy on the market that is sold as a mother dog with her puppies. Each mother dog comes with 2, 3, or 4 puppies. The number of puppies in the package remains a surprise until the toy is purchased and opened. Suppose the toy company has stated that one half of the toy packages contain 2 puppies, one third of the packages contain 3 puppies, and one sixth of the packages contain 4 puppies. Describe what could be used to perform a simulation for determining the probability of randomly receiving a certain number of puppies.

   **Sample answer:** A number cube could be used to simulate the random event of receiving 2, 3, or 4 puppies. Let #1-3 represent receiving 2 puppies; let #4 and 5 represent receiving 3 puppies; let #6 represent receiving 4 puppies.

3. **AUTOMOBILES** A consumer group surveyed its members and found that many of them had flats or blowouts with a certain brand of tire. Out of a total of 20,224 tires purchased, 984 developed problems within the first 1000 miles. Lea has just had one of these tires installed on her car. What is the probability that her tire will have a flat or blowout in the first 1000 miles? **about 4.9%**

4. **POLYGRAPH TESTING** A former FBI detective has developed a voice stress analysis device to determine whether or not a person is telling the truth. He claims that the device is accurate 95% of the time. Additionally, a traditional polygraph machine has a reported accuracy rate of 80%.

   a. Suppose four criminal suspects are given the voice stress analysis. According to the reported empirical probability, what is the probability that they can correctly analyze the accuracy of all four suspects’ statements? Round your answer to the nearest tenth of a percent. **81.5%**

   b. If a randomly chosen person is given both the voice stress analysis and the traditional polygraph to validate his or her statements, what is the probability that both devices are able to correctly determine the accuracy of the person’s statements? **76%**
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