Anticipation Guide

9
Quadratic and Exponential Functions

A1

Step 1
Before you begin Chapter 9

• Read each statement.
• Decide whether you Agree (A) or Disagree (D) with the statement.
• Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1</th>
<th>A, D, or NS</th>
<th>Statement</th>
<th>STEP 2</th>
<th>A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>A</td>
<td>1. The graph of a quadratic function is a parabola.</td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>2.</td>
<td>D</td>
<td>2. The graph of (4x^2 - 2x + 7) will be a parabola opening downward since the coefficient of (x^2) is positive.</td>
<td></td>
<td>D</td>
</tr>
<tr>
<td>3.</td>
<td>D</td>
<td>3. A quadratic function’s axis of symmetry is either the (x)-axis or the (y)-axis.</td>
<td></td>
<td>D</td>
</tr>
<tr>
<td>4.</td>
<td>A</td>
<td>4. The graph of a quadratic function opening upward has no maximum value.</td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>5.</td>
<td>A</td>
<td>5. The (x)-intercepts of the graph of a quadratic function are the solutions to the related quadratic equation.</td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>6.</td>
<td>D</td>
<td>6. All quadratic equations have two real solutions.</td>
<td></td>
<td>D</td>
</tr>
<tr>
<td>7.</td>
<td>A</td>
<td>7. Any quadratic expression can be written as a perfect square by a method called completing the square.</td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>8.</td>
<td>D</td>
<td>8. The quadratic formula can only be used to solve quadratic equations that cannot be solved by factoring or graphing.</td>
<td></td>
<td>D</td>
</tr>
<tr>
<td>9.</td>
<td>D</td>
<td>9. A function containing powers is called an exponential function.</td>
<td></td>
<td>D</td>
</tr>
<tr>
<td>10.</td>
<td>A</td>
<td>10. Receiving compound interest on a bank account is one example of exponential growth.</td>
<td></td>
<td>A</td>
</tr>
</tbody>
</table>

Step 2
After you complete Chapter 9

• Reread each statement and complete the last column by entering an A or a D.
• Did any of your opinions about the statements change from the first column?
• For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.

Study Guide and Intervention

9-1
Graphing Quadratic Functions

Characteristics of Quadratic Functions

A function described by an equation of the form \(f(x) = ax^2 + bx + c\), where \(a \neq 0\).

The parent graph of the family of quadratic functions is \(y = x^2\). Graphs of quadratic functions have a general shape called a parabola. A parabola opens upward and has a minimum point when the value of \(a\) is positive, and a parabola opens downward and has a maximum point when the value of \(a\) is negative.

Example 1

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Graph the ordered pairs in the table and connect them with a smooth curve.

Example 2

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-7</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-7</td>
</tr>
</tbody>
</table>

Graph the ordered pairs in the table and connect them with a smooth curve.

Exercises

Use a table of values to graph each function. Determine the domain and range.

1. \(y = x^2 + 2\)
2. \(y = -x^2 - 4\)
3. \(y = x^2 - 3x + 2\)

D: \(\{x | x\) is a real number\} \ R: \{y | y \geq 2\}

D: \(\{x | x\) is a real number\} \ R: \{y | y \leq -4\}

D: \(\{x | x\) is a real number\} \ R: \{y | y \geq -\frac{1}{4}\}
**Graphing Quadratic Functions**

Parabolas have a geometric property called symmetry. That is, if the figure is folded in half, each half will match the other half exactly. The vertical line containing the fold line is called the axis of symmetry. The axis of symmetry contains the minimum or maximum point of the parabola, the vertex.

#### Example

Consider the graph of $y = 2x^2 + 4x + 1$.

a. Write the equation of the axis of symmetry.

In $y = 2x^2 + 4x + 1$, $a = 2$ and $b = 4$. Substitute these values into the equation of the axis of symmetry.

$$ x = \frac{-b}{2a} $$

$$ x = \frac{-4}{2(2)} $$

The axis of symmetry is $x = -1$.

c. Identify the vertex as a maximum or a minimum.

Since the coefficient of the $x^2$-term is positive, the parabola opens upward, and the vertex is a minimum point.

d. Graph the function.

**Exercises**

Consider each equation. Determine whether the function has **maximum** or **minimum** value. Find the equation of the axis of symmetry. Graph the function.

1. $y = x^2 + 3$
   - **min:** (0, 3);
   - **D:** $\{x \mid x \text{ all reals}\}$;
   - **R:** $\{y \mid y \geq 3\}; x = 0$

2. $y = -x^2 - 4x - 4$
   - **max:** (-2, 0);
   - **D:** $\{x \mid x \text{ all reals}\}$;
   - **R:** $\{y \mid y \leq 0\}; x = -2$

3. $y = x^2 + 2x + 3$
   - **min:** (-1, 2);
   - **D:** $\{x \mid x \text{ all reals}\}$;
   - **R:** $\{y \mid y \geq 2\}; x = -1$

**Answers**

1. $y = x^2 - 4$
   - **D:** all reals;
   - **R:** $\{y \mid y \geq -4\}$

2. $y = -x^2 + 3$
   - **D:** all reals;
   - **R:** $\{y \mid y \leq 3\}$

3. $y = x^2 - 2x - 6$
   - **D:** all reals;
   - **R:** $\{y \mid y \geq -7\}$

4. $y = 2x^2 - 8x + 6$
   - **D:** all reals;
   - **R:** $\{y \mid y \leq 0\}$(2, -2); $x = 2$; (0, 6)

5. $y = x^2 + 4x + 6$
   - **D:** all reals;
   - **R:** $\{y \mid y \geq -4\}$(-2, 2); $x = -2$; (0, 6)

6. $y = -3x^2 - 12x + 3$
   - **D:** all reals;
   - **R:** $\{y \mid y \geq -6\}(-2, 15); x = -2$; (0, 3)

**Exercises**

Consider each equation.

a. Determine whether the function has **maximum** or **minimum** value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

7. $y = 2x^2$
   - **min:** (0, 0);
   - **D:** all reals;
   - **R:** $\{y \mid y \geq 0\}$

8. $y = x^2 - 2x - 3$
   - **max:** (0, -3);
   - **D:** all reals;
   - **R:** $\{y \mid y \leq -3\}$

9. $y = -x^2 + 4x - 1$
   - **min:** (0, 0);
   - **D:** all reals;
   - **R:** $\{y \mid y \leq 0\}$

Graph each function.

10. $f(x) = -x^2 - 2x + 2$
   - **D:** all reals;
   - **R:** $\{y \mid y \geq 2\}$

11. $f(x) = 2x^2 + 4x - 2$
   - **D:** all reals;
   - **R:** $\{y \mid y \leq -2\}$

12. $f(x) = -2x^2 - 4x + 6$
   - **D:** all reals;
   - **R:** $\{y \mid y \leq 6\}$
9-1 Practice

Graphing Quadratic Functions

Use a table of values to graph each function. Determine the domain and range.

1. \( y = -x^2 + 2 \)
   - Domain: all reals
   - Range: \( y \leq 2 \)

2. \( y = x^2 - 6x + 3 \)
   - Domain: all reals
   - Range: \( y \geq 3 \)

3. \( y = -2x^2 - 8x - 5 \)
   - Domain: all reals
   - Range: \( y \geq -5 \)

Find the vertex, the equation of the axis of symmetry, and the \( y \)-intercept.

4. \( y = x^2 - 9 \)
   - Vertex: \((0, -9)\)
   - Axis of symmetry: \( x = 0 \)
   - \( y \)-intercept: \((0, -9)\)

5. \( y = -2x^2 + 8x - 5 \)
   - Vertex: \((2, 3)\)
   - Axis of symmetry: \( x = 2 \)
   - \( y \)-intercept: \((0, -5)\)

Consider each equation. Determine whether the function has maximum or minimum value. State the maximum or minimum value. What are the domain and range of the function?

6. \( y = 5x^2 - 2x + 2 \)
   - Minimum: \((0.2, 1.8)\)
   - Domain: all reals
   - Range: \( y \geq 1.8 \)

Graph each function.

7. \( f(x) = -x^2 + 3 \)

8. \( f(x) = -2x^2 + 8x - 3 \)

9. \( f(x) = 2x^2 + 8x + 1 \)

10. \( f(x) = -x^2 + 3 \)

11. \( f(x) = -2x^2 + 8x - 3 \)

12. \( f(x) = 2x^2 + 8x + 1 \)

13. BASEBALL A player hits a baseball into the outfield. The equation \( h = -0.009x^2 + x + 3 \) gives the path of the ball, where \( h \) is the height and \( x \) is the horizontal distance the ball travels.
   - a. What is the equation of the axis of symmetry? \( x = 100 \)
   - b. What is the maximum height reached by the baseball? 53 ft
   - c. An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it? 200 ft

Chapter 9 8

Answers (Lesson 9-1)
Graphing Cubic Functions

A cubic function is a polynomial written in the form of $f(x) = ax^3 + bx^2 + cx + n$, where $a \neq 0$. Cubic functions do not have absolute minimum and maximum values like quadratic functions do, but they can have a local minimum and a local maximum point.

Parent Function: $f(x) = x^3$

Domain: all real numbers
Range: all real numbers

Example Use a table of values to graph $y = x^3 + 3x^2 - 1$. Then use the graph to estimate the locations of the local minimum and local maximum points.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-1</td>
<td>2</td>
<td>2</td>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The "S" shape extends to infinity in the positive $y$ direction and to negative infinity in the negative $y$ direction.

The local minimum is located at (0, –1). The local maximum is located at (–2, 2).

Exercises

Use a table of values to graph each equation. Then use the graph to estimate the locations of the local minimum and local maximum points.

1. $y = 0.5x^3 + x^2 - 1$
   - local maximum: (–1.3, –0.4); local minimum: (0.1, 0)
2. $y = -2x^3 - 3x^2 - 1$
   - local maximum: (0, 1); local minimum: (–1, –2)
3. $y = x^3 + 3x^2 + x - 4$
   - local maximum: (–1.8, –1.9); local minimum: (–0.2, –4.1)

Solve each equation by graphing.

1. $x^2 + 7x + 12 = 0$
   - no real roots
2. $x^2 - x - 12 = 0$
   - $x = -3, -4$
3. $x^2 - 4x + 5 = 0$
   - no real roots

Example 1 Solve $x^3 + 4x + 3 = 0$ by graphing.
Graph the related function $f(x) = x^3 + 4x + 3$.
The equation of the axis of symmetry is $x = -rac{4}{2} = -2$. The vertex is at (–2, –1). Graph the vertex and several other points on either side of the axis of symmetry.

Example 2 Solve $x^3 - 6x + 9 = 0$ by graphing.
Graph the related function $f(x) = x^3 - 6x + 9$.
The equation of the axis of symmetry is $x = rac{6}{3} = 2$. The vertex is at (2, 0). Graph the vertex and several other points on either side of the axis of symmetry.
Estimate Solutions
The roots of a quadratic equation may not be integers. If exact roots cannot be found, they can be estimated by finding the consecutive integers between which the roots lie.

Example
Solve \(x^2 + 6x + 6 = 0\) by graphing. If integral roots cannot be found, estimate the roots by stating the consecutive integers between which the roots lie.

Graph the related function \(f(x) = x^2 + 6x + 6\).

Notice that the value of the function changes from negative to positive between the \(x\)-values of \(-5\) and \(-4\) and between \(-2\) and \(-1\).

The \(x\)-intercepts of the graph are between \(-5\) and \(-4\) and between \(-2\) and \(-1\).

Exercises
Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

1. \(x^2 - 2x + 3 = 0\)
2. \(c^2 + 6c + 8 = 0\)
3. \(a^2 - 2a = -1\)
4. \(n^2 - 7n = -10\)
5. \(p^2 + 4p + 2 = 0\)
6. \(x^2 + x - 3 = 0\)
7. \(d^2 + 6d = -3\)
8. \(h^2 + 1 = 4h\)
9-2  Practice
Solving Quadratic Equations by Graphing

Solve each equation by graphing.

1. \(x^2 - 5x + 6 = 0\)  \(2, 3\)
2. \(2m^2 + 6m + 9 = 0\)  \(-3\)
3. \(b^2 - 3b + 4 = 0\)  \(\emptyset\)
4. \(p^2 + 4p = 3\)  \((-4.6, 0.6)\)
5. \(2n^2 + 5 = 10m\)  \((0.6, 4.4)\)
6. \(2v^2 + 8v = -7\)  \((-2.7, -1.3)\)

Solve each equation by graphing. If integral roots cannot be found, estimate the roots to the nearest tenth.

7. NUMBER THEORY Two numbers have a sum of 2 and a product of -8. The quadratic equation \(-x^2 + 2n + 8 = 0\) can be used to determine the two numbers.
   a. Graph the related function \(f(n) = -n^2 + 2n + 8\) and determine its \(x\)-intercepts. \(-2, 4\)
   b. What are the two numbers? \(-2\) and 4

8. DESIGN A footbridge is suspended from a parabolic support. The function \(h(x) = -\frac{1}{30}x^2 + 9\) represents the height in feet of the support above the walkway, where \(x = 0\) represents the midpoint of the bridge.
   a. Graph the function and determine its \(x\)-intercepts. \(-15, 15\)
   b. What is the length of the walkway between the two supports? \(30\) ft

9-2  Word Problem Practice
Solving Quadratic Equations by Graphing

1. FARMING In order for Ray to decide how much fertilizer to apply to his corn crop this year, he reviews records from previous years. He finds that his crop yield depends on the amount of fertilizer he applies to his fields according to the equation \(y = -x^2 + 4x + 12\). Graph the function, and find the point at which Ray gets the highest yield possible.

2. LIGHT Ayzha and Jeremy hold a flashlight so that the light falls on a piece of graph paper in the shape of a parabola. Ayzha and Jeremy sketch the shape of the light beam and find that the equation \(y = x^2 - 3x - 10\) matches the shape of the light beam. Determine the roots of the function. \(-2\) and 5

3. FRAMING A rectangular photograph is 7 inches long and 6 inches wide. The photograph is framed using material \(x\) inches wide. If the area of the frame and photograph combined is 156 square inches, what is the width of the framing material? \(3\) in. 

4. WRAPPING PAPER Can a rectangular piece of wrapping paper with an area of 81 square inches have a perimeter of 60 inches? (Hint: Let length = \(30 - w\)) Explain. Solving the equation \((30 - w)w = 81\) gives \(w = 3\) or 27. A 3 in. by 27 in. sheet of paper would work.

5. ENGINEERING The shape of a satellite dish is often parabolic because of the reflective qualities of parabolas. Suppose a particular satellite dish is modeled by the following equation. \(0.5x^2 = 2y\)

   a. Approximate the solution by graphing. \(-2\) and 2
   b. On the coordinate plane above, translate the parabola so that there is only one root. Label this curve A. See students' work.
   c. Translate the parabola so that there are no roots. Label this curve B. See students' work.
Parabolas Through Three Given Points

If you know two points on a straight line, you can find the equation of the line. To find the equation of a parabola, you need three points on the curve.

Here is how to approximate an equation of the parabola through the points \((0, -2), (3, 0),\) and \((5, 2)\).

Use the general equation \(y = ax^2 + bx + c\). By substituting the given values for \(x\) and \(y\), you get three equations.

\[
\begin{align*}
(0, -2): & \quad -2 = c \\
(3, 0): & \quad 0 = 9a + 3b + c \\
(5, 2): & \quad 2 = 25a + 5b + c
\end{align*}
\]

First, substitute \(-2\) for \(c\) in the second and third equations. Then solve those two equations as you would any system of two equations. Multiply the second equation by 5 and the third equation by \(-3\).

\[
\begin{align*}
0 &= 9a + 3b - 10 \\
0 &= 45a + 15b - 10 \\
0 &= 25a + 5b + 6 \\
6 &= -75a + 150 - 6 \\
-6 &= -30a - 4
\end{align*}
\]

To find \(b\), substitute \(\frac{1}{15}\) for \(a\) in either the second or third equation.

\[
0 = 9 \cdot \frac{1}{15} + 3b - 2
\]

\[
b = \frac{7}{15}
\]

The equation of a parabola through the three points is

\[
y = \frac{1}{15} x^2 + \frac{7}{15} x - 2.
\]

Find the equation of a parabola through each set of three points.

1. \((1, 5), (0, 6), (2, 3)\)
   \[
y = \frac{1}{2} x^2 - \frac{1}{2} x + 6
\]

2. \((-5, 0), (0, 0), (8, 100)\)
   \[
y = \frac{25}{26} x^2 + \frac{125}{26} x
\]

3. \((-4, -6), (0, 1), (3, -2)\)
   \[
y = -\frac{3}{5} x^2 + \frac{18}{5} x
\]

4. \((1, 3), (6, 0), (0, 0)\)
   \[
y = \frac{1}{2} x^2 + 1\]

5. \((-2, 2), (5, -3), (0, -1)\)
   \[
y = -\frac{1}{4} x^2 - \frac{1}{4} x + 1
\]

6. \((0, 4), (4, 0), (-4, 4)\)
   \[
y = \frac{1}{8} x^2 - \frac{1}{2} x + 4
\]

7. \((-2, 3), (5, -1), (0, -1)\)
   \[
y = \frac{18}{30} x^2 + \frac{83}{30} x - 1
\]

8. \((-8, 5), (3, 2), (0, -1)\)
   \[
y = -\frac{1}{2} x^2 + \frac{1}{2} x + 4
\]
Describe how the graph of each function is related to the graph of \(f(x) = x^2\).

**Example**

Describe how the graph of each function is related to the graph of \(f(x) = x^2\).

a. \(g(x) = 2x^2\)
   The function can be written as \(f(x) = ax^2\) where \(a = 2\). Because \(|a| > 1\), the graph of \(y = 2x^2\) is the graph of \(y = x^2\) that is stretched vertically.

b. \(h(x) = -\frac{1}{2}x^2 - 3\)
   The negative sign causes a reflection across the \(x\)-axis. Then a dilation occurs in which \(a = \frac{1}{2}\) and a translation in which \(c = -3\). So the graph of \(y = -\frac{1}{2}x^2 - 3\) is reflected across the \(x\)-axis, dilated wider than the graph of \(f(x) = x^2\), and translated down 3 units.

**Exercises**

Describe how the graph of each function is related to the graph of \(f(x) = x^2\).

1. \(h(x) = -5x^2\)
   Compression of \(y = x^2\) narrower than the graph of \(f(x) = x^2\) reflected over the \(x\)-axis.

2. \(g(x) = -x^2 + 1\)
   Translation of \(y = x^2\) reflected over the \(x\)-axis and up 1 unit.

3. \(h(x) = -\frac{1}{4}x^2 - 1\)
   Dilation of \(y = x^2\) wider than the graph of \(f(x) = x^2\) reflected over the \(x\)-axis translated down 1 unit.

4. \(g(x) = x^2 + 2\)
   Translation of \(y = x^2\) up 2 units.

5. \(h(x) = -1 + x^2\)
   Translation of \(y = x^2\) down 1 unit.

6. \(g(x) = -\frac{1}{2}x^2\)
   Compression of \(y = x^2\) narrower than the graph of \(f(x) = x^2\)

7. \(h(x) = 7x^2\)
   Dilation of \(y = x^2\) wider than the graph of \(f(x) = x^2\)

8. \(g(x) = -\frac{1}{3}x^2\)
   Compression of \(y = x^2\) narrower than the graph of \(f(x) = x^2\)

9. \(h(x) = 5 - \frac{1}{2}x^2\)
   Reflected over the \(x\)-axis and up 5 units

10. \(g(x) = 4x^2 + 1\)
    Compression of \(y = x^2\) narrower than the graph of \(f(x) = x^2\) translated up 1 unit

Match each equation to its graph.

11. \(y = \frac{1}{2}x^2 - 2\) B A.

12. \(y = -\frac{1}{2}x^2 + 2\) C.

13. \(y = -2x^2 + 2\) A.

14. \(y = -x^2 + 2\) D.
9-3 Practice

Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of \( f(x) = x^2 \).

1. \( g(x) = 10 + x^2 \)
   - Translation of \( y = x^2 \) up 10 units.

2. \( h(x) = -\frac{3}{5} + x^2 \)
   - Reflection of \( y = x^2 \) across the \( x \)-axis, translated up \( \frac{3}{5} \) unit.

3. \( g(x) = 9 - x^2 \)
   - Reflection of \( y = x^2 \) across the \( x \)-axis, translated up 9 units.

4. \( h(x) = 2x^2 + 2 \)
   - Compression of \( y = x^2 \) narrower than the graph of \( f(x) = x^2 \), translated up 2 units.

5. \( g(x) = -\frac{3}{5}x^2 - \frac{3}{5} \)
   - Dilation of \( y = x^2 \) wider than the graph of \( f(x) = x^2 \), reflected over the \( x \)-axis, translated down \( \frac{3}{5} \) unit.

6. \( h(x) = 4 - 3x^2 \)
   - Compression of \( y = x^2 \) narrower than the graph of \( f(x) = x^2 \), reflected over the \( x \)-axis, translated up 4 units.

Match each equation to its graph.

A. \( y = -3x^2 - 1 \)
B. \( y = -\frac{3}{5}x^2 + 1 \)
C. \( y = 3x^2 + 1 \)

List the functions in order from the most vertically stretched to the least vertically stretched graph.

10. \( f(x) = 3x^2, g(x) = \frac{1}{2}x^2, h(x) = -2x^2 \)
    - \( f(x), g(x), h(x) \)

11. \( f(x) = \frac{1}{2}x^2, g(x) = -\frac{1}{5}x^2, h(x) = 4x^2 \)
    - \( h(x), f(x), g(x) \)

12. PARACHUTING
   - Two parachutists jump from two different planes as part of an aerial show. The height \( h_1 \) of the first parachutist in feet after \( t \) seconds is modeled by the function \( h_1 = -10t^2 + 5000 \). The height \( h_2 \) of the second parachutist in feet after \( t \) seconds is modeled by the function \( h_2 = -16t^2 + 4000 \). \( h = f^2 \)
   - a. What is the parent function of the two functions given?
   - b. Describe the transformations needed to obtain the graph of \( h_1 \) from the parent function. Compression of \( y = x^2 \) narrower than the graph of \( f(x) = x^2 \), reflected over the \( x \)-axis, translated up 5000 units.
   - c. Which parachutist will reach the ground first? The second parachutist

9-3 Word Problem Practice

Transformations of Quadratic Functions

1. SPRINGS
   - The potential energy stored in a spring is modeled by the function \( U = \frac{1}{2}kx^2 \) where \( k \) is a constant known as the spring constant, and \( x \) is the distance the spring is stretched or compressed from its initial position. Explain how the graph of the function for a spring where \( k = 2 \) newtons/meter differs from the graph of the function for a spring where \( k = 10 \) newtons/meter.
   - a. Graph and label each function on the same coordinate plane.
   - b. Explain how each graph is related to the graph of \( f(x) = x^2 \).
   - c. After how many seconds will the first car pass the second car?

4. ACCELERATION
   - The distance \( d \) in feet a car accelerating at \( 6 \) ft/s\(^2 \) travels after \( t \) seconds is modeled by the function \( d = 3t^2 \). Suppose that at the same time the first car begins accelerating, a second car begins accelerating at \( 4 \) ft/s\(^2 \) exactly 100 feet down the road from the first car.
   - a. Describe how the graph of each function is related to the graph of \( f(x) = x^2 \).
   - b. Explain how each graph is related to the graph of \( f(x) = x^2 \).
   - c. After how many seconds will the first car pass the second car?

   10 seconds
Translating Quadratic Graphs

When a figure is moved to a new position without undergoing any rotation, then the figure is said to have been translated to that position.

The graph of a quadratic equation in the form \(y = (x - b)^2 + c\) is a translation of the graph of \(y = x^2\).

1. Start with \(y = x^2\).
2. Slide to the right 4 units. \(y = (x - 4)^2\)
3. Then slide up 3 units. \(y = (x - 4)^2 + 3\)

These equations have the form \(y = (x - b)^2 + c\). Graph each equation.

4. \(y = (x - 1)^2\)
5. \(y = (x - 3)^2\)
6. \(y = (x + 2)^2\)

These equations have the form \(y = (x - b)^2 + c\). Graph each equation.

7. \(y = (x - 2)^2 + 1\)
8. \(y = (x - 1)^2 + 2\)
9. \(y = (x + 1)^2 - 2\)

---

Solving Quadratic Equations by Completing the Square

Complete the Square Perfect square trinomials can be solved quickly by taking the square root of both sides of the equation. A quadratic equation that is not in perfect square form can be made into a perfect square by a method called completing the square.

Completing the Square

To complete the square for any quadratic equation of the form \(x^2 + bx + c\):

1. Find one-half of \(b\), the coefficient of \(x\).
2. Square the result in Step 1.
3. Add the result of Step 2 to \(x^2 + bx + c\).

\[
x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2
\]

Find the value of \(c\) that makes \(x^2 + 2x + c\) a perfect square trinomial.

1. \(x^2 + 10x + c\)
2. \(x^2 + 14x + c\)
3. \(x^2 - 4x + c\)
4. \(x^2 - 8x + c\)
5. \(x^2 + 5x + c\)
6. \(x^2 + 3x + c\)
7. \(x^2 - 3x + c\)
8. \(x^2 - 15x + c\)
9. \(x^2 + 28x + c\)

Exercises

Find the value of \(c\) that makes each trinomial a perfect square.

1. \(x^2 + 10x + c\)
2. \(x^2 + 14x + c\)
3. \(x^2 - 4x + c\)
4. \(x^2 - 8x + c\)
5. \(x^2 + 5x + c\)
6. \(x^2 + 3x + c\)
7. \(x^2 - 3x + c\)
8. \(x^2 - 15x + c\)
9. \(x^2 + 28x + c\)
10. \(x + 22x + c\)
### 9-4 Study Guide and Intervention (continued)

**Solving Quadratic Equations by Completing the Square**

Since few quadratic expressions are perfect square trinomials, the method of completing the square can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

**Step 1** Find $\frac{b}{2a}$.

**Step 2** Add $\left(\frac{b}{2a}\right)^2$ to both sides.

**Step 3** Solve each equation by completing the square. Round to the nearest tenth if necessary.

#### Example

Solve $x^2 + 6x + 3 = 10$ by completing the square.

- $x^2 + 6x + 3 = 10$ Original equation
- $x^2 + 6x + 9 = 7$ Subtract 3 from each side.
- $(x + 3)^2 = 7$ Simplify.
- $x + 3 = \pm \sqrt{7}$ Take the square root of each side.
- $x = -3 + 4$ or $x = -3 - 4$
- $x = 1$ or $x = -7$

The solution set is $\{-7, 1\}$.

#### Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 - 4x + 3 = 0$</td>
<td>$1, -3$</td>
</tr>
<tr>
<td>$x^2 + 6x + 2 = 0$</td>
<td>$-2, 5$</td>
</tr>
<tr>
<td>$x^2 + 10x = -9$</td>
<td>$-1, -9$</td>
</tr>
<tr>
<td>$x^2 - 4x - 5 = 0$</td>
<td>$-2, 5$</td>
</tr>
<tr>
<td>$x^2 - 2x + 1 = 0$</td>
<td>$-1, 2$</td>
</tr>
<tr>
<td>$x^2 + 20x + 11 = 0$</td>
<td>$-10, -1$</td>
</tr>
<tr>
<td>$x^2 - 2x + 1 = 0$</td>
<td>$-1, 2$</td>
</tr>
<tr>
<td>$x^2 + 10x + 1 = 0$</td>
<td>$-1, 2$</td>
</tr>
<tr>
<td>$x^2 - 10x + 24 = 14$</td>
<td>$-12, 2$</td>
</tr>
<tr>
<td>$x^2 - 18x + 19 = 15$</td>
<td>$-12, 2$</td>
</tr>
<tr>
<td>$x^2 + 16x = -16$</td>
<td>$-12, 2$</td>
</tr>
<tr>
<td>$x^2 + 4x + 2 = 8$</td>
<td>$-2, 3$</td>
</tr>
<tr>
<td>$x^2 + 4x + 4 = 0$</td>
<td>$-2, 3$</td>
</tr>
</tbody>
</table>

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### 9-4 Skills Practice

**Solving Quadratic Equations by Completing the Square**

Find the value of $c$ that makes each trinomial a perfect square.

1. $x^2 + 6x + c = 9$
2. $x^2 + 4x + c = 4$
3. $x^2 - 14x + c = 49$
4. $x^2 - 2x + c = 1$
5. $x^2 - 18x + c = 81$
6. $x^2 + 20x + c = 100$
7. $x^2 + 5x + c = 6.25$
8. $x^2 - 70x + c = 1225$
9. $x^2 - 11x + c = 30.25$
10. $x^2 + 9x + c = 20.25$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

11. $x^2 + 4x - 12 = 0$ $2, -6$
12. $x^2 - 8x + 15 = 0$ $3, 5$
13. $x^2 + 6x = 7$ $-7, 1$
14. $x^2 - 2x = 15$ $-3, 5$
15. $x^2 - 14x + 30 = 6$ $2, 12$
16. $x^2 - 12x + 21 = 10$ $-11, -1$
17. $x^2 + 4x + 1 = 0$ $0.3, 3.7$
18. $x^2 + 6x + 4 = 0$ $0.8, 5.2$
19. $x^2 - 8x + 10 = 0$ $1.6, 6.4$
20. $x^2 - 2x = 5$ $-1.4, 3.4$
21. $2x^2 + 20x = -2$ $-9.9, -0.1$
22. $0.5x^2 + 8x = -7$ $-15.1, -0.9$
9-4 Practice
Solving Quadratic Equations by Completing the Square

Find the value of \( c \) that makes each trinomial a perfect square.

1. \( x^2 - 24x + 144 \)
2. \( x^2 + 28x + 196 \)
3. \( x^2 + 40x + 400 \)
4. \( x^2 + 9x + 9 \) \( \frac{9}{4} \)
5. \( x^2 - 9x + \frac{81}{4} \)
6. \( x^2 - x + \frac{1}{4} \)

Solve each equation by completing the square. Round to the nearest tenth if necessary.

7. \( x^2 - 14x + 24 = 0 \)
8. \( x^2 + 12x = 13 \)
9. \( x^2 - 30x + 56 = -25 \)
10. \( x^2 + 8x + 9 = 0 \)
11. \( x^2 - 10x + 6 = -7 \)
12. \( x^2 + 18x + 50 = 9 \)
13. \(-6x^2 + 15x - 3 = 0 \)
14. \( 4x^2 - 72 = 24x \)
15. \( 0.9x^2 + 5x - 4 = 0 \)
16. \( 0.4x^2 + 0.8x - 0.2 \)
17. \( \frac{1}{2}x^2 - x - 10 = 0 \)
18. \( \frac{1}{2}x^2 + x - 2 = 0 \)
19. \(-2.2, 0.2 \)
20. \(-2.2, 8.2 \)
21. \(-6.2, 2.3 \)
22. \(-6.2, 2.3 \)
23. \( x^2 + 10x + \underline{_____} = (x + \underline{_____})^2 \)

9-4 Word Problem Practice
Solving Quadratic Equations by Completing the Square

1. INTERIOR DESIGN Modular carpeting is installed in small pieces rather than as a large roll so that only a few pieces need to be replaced if a small area is damaged. Suppose the room shown in the diagram below is being fitted with modular carpeting. Complete the squares to determine the number of 1 ft by 1 ft squares of carpeting needed to finish the room. Fill in the missing terms in the corresponding equation below. 25; 5

![Diagram of room]

2. FALLING OBJECTS Keisha throws a rock down an old well. The distance \( h \) (in feet) the rock falls after \( t \) seconds can be represented by the equation \( h = -16t^2 + 32t + 256 \). To the nearest tenth of a second, how long does it take for the rock to hit the water? 1 second

3. MARS On Mars, the gravity acting on an object is less than that on Earth. On Earth, a golf ball hit with an initial upward velocity of 26 meters per second will reach a maximum height of about 34.5 meters. The height \( h \) of an object on Mars that leaves the ground with an initial velocity of 26 meters per second is given by the equation \( h = -1.9t^2 + 29t \). Find the maximum height if the same golf ball is hit on Mars. Round your answer to the nearest tenth. 88.9 m

4. FROGS A frog sitting on a stump 3 feet high hops off and lands on the ground. During its leap, its height \( h \) (in feet) is given by \( h = -0.5t^2 + 2t + 3 \), where \( d \) is the distance from the base of the stump. How far is the frog from the base of the stump when it landed on the ground? \( 2 + \sqrt{10} \) or about 5.16 ft

5. GARDENING Peg is planning a rectangular vegetable garden using 250 feet of fencing material. She only needs to fence three sides of the garden since one side borders an existing fence. Answers

- Let \( x \) be the width of the rectangle. Write an expression to represent the area of the garden if she uses all the fencing material.
- If the short side is 62.5 ft and the other side 125 ft, the garden will be the largest possible, with an area of 7812.5 ft\(^2\).
9-4 Enrichment
Factoring Quartic Polynomials

Completing the square is a useful tool for factoring and solving quadratic expressions. You can utilize a similar technique to factor simple quartic polynomials of the form \( ax^4 + bx^2 + c = 0 \).

**Example**
Factor the quartic polynomial \( x^4 + 64 \).

**Step 1**
Find the value of the middle term needed to complete the square.
This value is \( 2 \sqrt{64} \), or 16x².

**Step 2**
Rewrite the original polynomial in factorable form:
\[
(x^2 + 16x^2 + \left(\frac{16}{2}\right)^2) - 16x^2
\]

**Step 3**
Factor the polynomials: \( (x^2 + 16)^2 - (4x)^2 \).

**Step 4**
Rewrite using the difference of two squares: \( (x^2 + 16 + 4x)(x^2 + 16 - 4x) \).
The factored form of \( x^4 + 64 \) is \( (x^2 + 4x + 8)(x^2 - 4x + 8) \). This could then be factored further, if needed, to find the solutions to a quartic equation.

**Exercises**
Factor each quartic polynomial.

1. \( x^4 + 4 \) 
   \( (x^2 + 2x + 2)(x^2 - 2x + 2) \)

2. \( x^4 + 324 \) 
   \( (x^2 + 6x + 18)(x^2 - 6x + 18) \)

3. \( x^4 + 2500 \) 
   \( 4x^4 + 9604 \)

4. \( (x^2 + 10x + 50)(x^2 - 10x + 50) \) 
   \( (x^2 + 14x + 98)(x^2 - 14x + 98) \)

5. \( x^4 + 1024 \) 
   \( 6x^4 + 5184 \)

6. \( (x^2 + 8x + 32)(x^2 - 8x + 32) \) 
   \( (x^2 + 12x + 72)(x^2 - 12x + 72) \)

7. \( x^4 + 484 \) 
   \( 8x^4 + 9 \)

8. \( x^2 + x\sqrt{28} + 22)(x^2 - x\sqrt{28} + 22) \) 
   \( (x^2 + x\sqrt{9} + 9)(x^2 - x\sqrt{9} + 9) \)

9. \( x^4 + 144 \) 
   \( 10x^4 + 16384 \)

10. \( x^4 + 2x\sqrt{6} + 12)(x^2 - 2x\sqrt{6} + 12) \) 
    \( (x^2 + 16x + 128)(x^2 - 16x + 128) \)

11. Factor \( x^4 + x^2 \) to come up with a general rule for factoring quartic polynomials.
    \( (x^2 + x\sqrt{2} + \sqrt{c}) + (x^2 + x\sqrt{2} - \sqrt{c}) \)

**Answers**

9-5 Study Guide and Intervention
Solving Quadratic Equations by Using the Quadratic Formula

**Quadratic Formula**
To solve the standard form of the quadratic equation, \( ax^2 + bx + c = 0 \), use the Quadratic Formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
that gives the solutions of \( ax^2 + bx + c = 0 \), when \( a \neq 0 \).

**Example 1**
Solve \( x^2 + 2x = 3 \) by using the Quadratic Formula.

Rewrite the equation in standard form.
\[
x^2 + 2x - 3 = 0
\]
Subtract 3 from each side.
\[
x^2 + 2x - 3 = 0
\]
Simplify.
\[
x^2 + 2x - 3 = 0
\]
Now let \( a = 1 \), \( b = 2 \), and \( c = -3 \) in the Quadratic Formula.
\[
x = \frac{-2 \pm \sqrt{4 + 4(3)}}{2}
\]
\[
x = \frac{-2 \pm \sqrt{16}}{2}
\]
\[
x = 1, -3
\]
The solution set is \([-3, 1]\).

**Example 2**
Solve \( x^2 - 6x - 2 = 0 \) by using the Quadratic Formula. Round to the nearest tenth if necessary.

For this equation \( a = 1 \), \( b = -6 \), and \( c = -2 \).
\[
x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}
\]
\[
x = \frac{6 \pm \sqrt{44}}{2}
\]
\[
x = 3.6, -0.3
\]
The solution set is \([-0.3, 3.6]\).

**Exercises**
Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. \( x^2 - 3x + 2 = 0 \)
   \( 1, 2 \)

2. \( x^2 - 8x = -16 \)
   \( 4 \)

3. \( 16x^2 - 8x = -1, 4 \)
   \( 4 \)

4. \( 4x^2 + 5x = 6 \)
   \( -6, 1 \)

5. \( 3x^2 + 2x = 8 \)
   \( -2, 4 \)

6. \( 8.2x^2 - 8x - 5 = 0 \)
   \( -0.4, 1.4 \)

7. \(-4x^2 + 19x = 21 \)
   \( 3, 3 \)

8. \( 2.4x^2 + 6x = 5 \)
   \( -3.7, 0.7 \)

9. \( 4x^2 + 22x - 15 = 0 \)
   \( 5, 3 \)

10. \( 8x^2 - 4x = 24 \)
    \( 3, 2 \)

11. \( 2x^2 + 5x = 8 \)
    \( -3.6, 1.1 \)

12. \( 8.4x^2 + 9x - 4 = 0 \)
    \( -1.5, 0.3 \)

13. \( 2.3x^2 + 9x + 4 = 0 \)
    \( -4, 1 \)

14. \( 4.8x^2 + 13x + 2 = 0 \)
    \( -2, 0.5 \)
**Solving Quadratic Equations by Using the Quadratic Formula**

The Discriminant In the Quadratic Formula, \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \), the expression under the radical sign, \( b^2 - 4ac \), is called the **discriminant**. The discriminant can be used to determine the number of real roots for a quadratic equation.

- **Case 1**: \( b^2 - 4ac < 0 \) no real roots
- **Case 2**: \( b^2 - 4ac = 0 \) one real root
- **Case 3**: \( b^2 - 4ac > 0 \) two real roots

**Example**

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

a. \( 12x^2 + 5x = 4 \)

Write the equation in standard form.

\( 12x^2 + 5x - 4 = 0 \)

\( b^2 - 4ac = (5)^2 - 4(12)(-4) \)

\( = 25 + 192 \)

\( = 217 \)

Since the discriminant is positive, the equation has two real roots.

b. \( 2x^2 + 3x = 4 \)

\( 2x^2 + 3x - 4 = 0 \)

\( b^2 - 4ac = (3)^2 - 4(2)(-4) \)

\( = 9 + 32 \)

\( = 41 \)

Since the discriminant is positive, the equation has two real roots.

**Exercises**

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

1. \( x^2 + 2x + 3 = 0 \)
2. \( 3x^2 - 7x - 8 = 0 \)
3. \( 2x^2 - 10x - 9 = 0 \)
4. \( 40, 2 \) real roots \( 145, 2 \) real solutions \( 172, 2 \) real solutions
5. \( 6, 3x^2 - 13x = 10 \)
6. \( 6.6x^2 - 10x + 10 = 0 \)
7. \( 65, 2 \) real solutions \( 289, 2 \) real solutions \( -140, 0 \) real solutions
8. \( 7, 2x^2 - 20 = -x \)
9. \( 8, 6x^2 = -11x - 40 \)
10. \( 9, 9 - 18x + 9x^2 = 0 \)
11. \( 161, 2 \) real solutions \( -839, 0 \) real solutions \( 0, 1 \) real solution
12. \( 10, 12x^2 + 9 = -6x \)
13. \( -396, 0 \) real solutions \( 2916, 2 \) real solutions \( 0, 1 \) real solution
14. \( 13, 8x^2 + 9x = 2 \)
15. \( 14, 4x^2 - 4x + 4 = 3 \)
16. \( 15, 3x^2 - 18x = -14 \)
17. \( 145, 2 \) real solutions \( 0, 1 \) real solution \( 156, 2 \) real solutions
9-5 Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. \(x^2 + 2x - 3 = 0\) \(\quad -3, 1\)
2. \(2x^2 + 8x + 7 = 0\) \(\quad -7, -1\)
3. \(x^2 - 4x + 6 = 0\) \(\quad \emptyset\)
4. \(x^2 - 2x + 8 = 0\) \(\quad \emptyset\)
5. \(2x + 5x - 5 = 0\) \(\quad -5, \frac{1}{2}\)
6. \(2x^2 + 12x + 10 = 0\) \(\quad -5, -1\)
7. \(2x^2 - 9x = -12\) \(\quad \emptyset\)
8. \(2x^2 - 5x = 12\) \(\quad -\frac{1}{2}, 4\)
9. \(x^2 + x = x - \frac{1}{3}\) \(\quad 1\)

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

10. \(3x^2 - 1 = -8x - 2\) \(\quad 0.1, 0.6\)
11. \(4x^2 + 7x + 15 = -3\) \(\quad \emptyset\)
12. \(1.6x^2 + 2x + 2.5 = 0\) \(\quad \emptyset\)
13. \(4.5x^2 + 4x - 1.5 = 0\) \(\quad -1, 0.3\)
14. \(-2x^2 + 2x + 2 = 0\) \(\quad -3, -1\)
15. \(3x^2 - \frac{3}{4}x = \frac{1}{2}\) \(\quad -0.3\)

25. CONSTRUCTION A roofer tosses a piece of roofing tile from a roof onto the ground 20 feet below. He tosses the tile with an initial downward velocity of 10 feet per second.

a. Write an equation to find how long it takes the tile to hit the ground. Use the model for vertical motion, \(H = -16t^2 + vt + h\), where \(H\) is the height of an object after \(t\) seconds, \(v\) is the initial velocity, and \(h\) is the initial height. (Hint: The object is thrown down, so the initial velocity is negative.) \(H = -16t^2 - 10t + 30\)

b. How long does it take the tile to hit the ground? \(\approx 1.1\) s

26. PHYSICS Lupe tosses a ball up to Quyen, waiting at a third-story window, with an initial velocity of 30 feet per second. She releases the ball from a height of 6 feet. The equation \(h = -16t^2 + 30t + 6\) represents the height \(h\) of the ball after \(t\) seconds. If the ball must reach a height of 25 feet for Quyen to catch it, does the ball reach Quyen? Explain. (Hint: Substitute 25 for \(h\) and use the discriminant.) \(\quad \emptyset\); the discriminant, \(-316\), is negative, so there is no solution.

9-5 Word Problem Practice

Solving Quadratic Equations by Using the Quadratic Formula

1. BUSINESS Tanya runs a catering business. Based on her records, her weekly profit can be approximated by the function \(f(x) = x^2 + 2x - 37\), where \(x\) is the number of meals she caters. If \(f(x)\) is negative, it means that the business has lost money. What is the least number of meals that Tanya needs to cater in order to have a profit?

2. AERONAUTICS At liftoff, the space shuttle Discovery has a constant acceleration of 16.4 feet per second squared and an initial velocity of 1341 feet per second due to the rotation of Earth. The distance Discovery has traveled \(t\) seconds after liftoff is given by the equation \(H = 1341t + 8.2t^2\). How long after liftoff has Discovery traveled 40,000 feet? Round your answer to the nearest tenth.

25.8 seconds

3. ARCHITECTURE The Golden Ratio appears in the design of the Greek Parthenon because the width and height of the façade are related by the equation \(\frac{w}{h} = \frac{H}{W}\). If the height of a model of the Parthenon is 16 inches, what is its width? Round your answer to the nearest tenth.

25.9 in.

4. CRAFTS Madelyn cuts a 60-inch pipe cleaner into two unequal pieces, and then she uses each piece to make a square. The sum of the areas of the squares was 117 square inches. Let \(x\) be the length of one piece. Write and solve an equation to represent the situation and find the lengths of the two original pieces.

5. SITE DESIGN The town of Smallport plans to build a new water treatment plant on a rectangular piece of land 75 yards wide and 200 yards long. The buildings and facilities need to cover an area of 10,000 square yards. The town’s zoning board wants the site designer to allow as much room as possible between each edge of the site and the buildings and facilities. Let \(x\) represent the width of the border.

a. Use an equation similar to \(A = l \times w\) to represent the situation.

b. Write the equation in standard quadratic form.

\(4x^2 = 550x + 5000 = 0; 9.8\) and 127.7

c. What should be the width of the border? Round your answer to the nearest tenth.

9.79 yd
**Golden Rectangles**

A golden rectangle has the property that its sides satisfy the following proportion:

\[
\frac{a + b}{a} = \frac{a}{b}
\]

Two quadratic equations can be written from the proportion. These are sometimes called golden quadratic equations.

1. In the proportion, let \( a = 1 \). Use cross-multiplication to write a quadratic equation.

\[
b^2 + b - 1 = 0
\]

2. Solve the equation in Exercise 1 for \( b \).

\[
b = \frac{-1 + \sqrt{5}}{2}
\]

3. In the proportion, let \( b = 1 \). Write a quadratic equation in \( a \).

\[
a^2 - a - 1 = 0
\]

4. Solve the equation in Exercise 3 for \( a \).

\[
a = \frac{1 + \sqrt{5}}{2}
\]

5. Explain why \( \frac{1}{2}(\sqrt{5} + 1) \) and \( \frac{1}{2}(\sqrt{5} - 1) \) are called golden ratios.

They are the ratios of the sides in a golden rectangle. The first is the ratio of the long side to the short side; the second is short side: long side.

Another property of golden rectangles is that a square drawn inside a golden rectangle creates another, smaller golden rectangle.

In the design at the right, opposite vertices of each square have been connected with quarter of circles.

For example, the arc from point \( B \) to point \( C \) is created by putting the point of a compass at point \( A \). The radius of the arc is the length \( BA \).

6. On a separate sheet of paper, draw a larger version of the design. Start with a golden rectangle with a long side of 10 inches.

The short side should be about 6 3/16 inches.
Exponential Functions

Identify Exponential Behavior

It is sometimes useful to know if a set of data is exponential. One way to tell is to observe the shape of the graph. Another way is to observe the pattern in the set of data.

Determine whether the set of data shown below displays exponential behavior. Write yes or no. Explain why or why not.

Method 1: Look for a Pattern
The domain values increase by regular intervals of 2, while the range values have a common factor of \(-\frac{1}{2}\). Since the domain values increase by regular intervals and the range values have a common factor, the data are probably exponential.

Method 2: Graph the Data
The graph shows rapidly decreasing values of \(y\) as \(x\) increases. This is characteristic of exponential behavior.

Exercises

Determine whether the set of data shown below displays exponential behavior. Write yes or no. Explain why or why not.

1. \([x \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 0]
   y \quad 5 \quad 10 \quad 15 \quad 18\)
   No; the domain values are at regular intervals, and the range values have a common difference 5.

2. \([x \quad 0 \quad 1 \quad 2 \quad 3]
   y \quad 3 \quad 9 \quad 27 \quad 81\)
   Yes; the domain values are at regular intervals, and the range values have a common factor 3.

3. \([x \quad -1 \quad 1 \quad 3 \quad 5]
   y \quad 32 \quad 16 \quad 8 \quad 4\)
   Yes; the domain values are at regular intervals, and the range values have a common factor \(2^2\).

4. \([x \quad -1 \quad 0 \quad 1 \quad 2 \quad 3]
   y \quad 3 \quad 1 \quad 3 \quad 9 \quad 27\)
   No; the domain values are at regular intervals, but the range values do not change.

5. \([x \quad -5 \quad 0 \quad 5 \quad 10]
   y \quad 1 \quad 0.25 \quad 0.0625\)
   Yes; the domain values are at regular intervals, and the range values have a common factor 0.5.

6. \([x \quad 0 \quad 1 \quad 2 \quad 3 \quad 4]
   y \quad \frac{1}{9} \quad \frac{1}{3} \quad 1 \quad 3\)
   Yes; the domain values are at regular intervals, and the range values have a common factor \(\frac{1}{3}\).

Graph each function. Find the \(y\)-intercept, and state the domain and range. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

1. \(y = 2^x\); 2.2
   \(1; D = \text{all reals}, R = \{y \mid y > 0\}; 4.9\)

2. \(y = (\frac{1}{2})^x\); \(-1.6\)
   \(1; D = \text{all reals}, R = \{y \mid y > 0\}; 5.8\)

Determine whether the set of data shown below displays exponential behavior. Write yes or no. Explain why or why not.

5. \([x \quad 3 \quad 2 \quad 1 \quad 0]
   y \quad 9 \quad 12 \quad 15 \quad 18\)
   No; the domain values are at regular intervals, and the range values have a common factor 3.

6. \([x \quad 50 \quad 30 \quad 10 \quad 0]
   y \quad 90 \quad 70 \quad 50 \quad 30\)
   Yes; the domain values are at regular intervals and the range values have a common difference 20.
9-6 Practice

Exponential Functions

Graph each function. Find the y-intercept and state the domain and range. Then use the graph to determine the approximate value of the given expression. Use a calculator to confirm the value.

1. \( y = \frac{1}{10} \cdot 2^{x} \)
   
   \( y \)-intercept: 0
   Domain: \( x \in \mathbb{R} \)
   Range: \( y > 0 \)

2. \( y = 3 \cdot 3^{x} \)
   
   \( y \)-intercept: 3
   Domain: \( x \in \mathbb{R} \)
   Range: \( y > 0 \)

3. \( y = \left( \frac{1}{2} \right)^{x} \)
   
   \( y \)-intercept: 1
   Domain: \( x \in \mathbb{R} \)
   Range: \( y > 0 \)

Graph each function. Find the y-intercept, and state the domain and range.

4. \( y = 4(2^{x} - 1) \)
   
   \( y \)-intercept: 0
   Domain: \( x \in \mathbb{R} \)
   Range: \( y > 0 \)

5. \( y = 2^{x} - 1 \)
   
   \( y \)-intercept: 1
   Domain: \( x \in \mathbb{R} \)
   Range: \( y > 0 \)

6. \( y = 0.5(3^{x} - 3) \)
   
   \( y \)-intercept: 0
   Domain: \( x \in \mathbb{R} \)
   Range: \( y > 0 \)

Determine whether the set of data shown below displays exponential behavior. Write yes or no. Explain why or why not.

7. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
</tbody>
</table>

Yes; the domain values are at regular intervals and the range values have a common factor 2.

8. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>

No; the domain values are at regular intervals and the range values have a common difference 10.

9. LEARNING Ms. Klemperer told her English class that each week students tend to forget one sixth of the vocabulary words they learned the previous week. Suppose a student learns 60 words. The number of words remembered can be described by the function \( W(x) = 60 \left( \frac{5}{6} \right)^{x} \), where \( x \) is the number of weeks that pass. How many words will the student remember after 3 weeks? About 35

10. BIOLOGY Suppose a certain cell reproduces itself in four hours. If a lab researcher begins with 50 cells, how many cells will there be after one day, two days, and three days? (Hint: Use the exponential function \( y = 50 \cdot 2^{x} \))

   3200 cells; 204,800 cells; 13,107,200 cells

9-6 Word Problem Practice

Exponential Functions

1. WASTE Suppose the waste generated by nonrecycled paper and cardboard products is approximated by the following function.

\[ y = 1000(2^{x}) \]

Sketch the exponential function on the coordinate grid below.

2. MONEY Tatyana's grandfather gave her one penny on the day she was born. He plans to double the amount he gives her every day. Estimate how much she will receive from her grandfather on the 12th day of her life.

About $20

3. PICTURE FRAMES Since a picture frame includes a border, the picture must be smaller in area than the entire frame. The table shows the relationship between picture area and frame length for a particular line of frames. Is this an exponential relationship? Explain.

No; there is no common factor between the picture areas.

4. DEPRECIATION The value of Royce Company's computer equipment is decreasing in value according to the following function.

\[ y = 4000(0.87)^{x} \]

In the equation, \( x \) is the number of years that have elapsed since the equipment was purchased and \( y \) is in dollars. What was the value 5 years after it was purchased? Round your answer to the nearest dollar.

$1994

5. METEOROLOGY The atmospheric pressure (in millibars) at a given altitude \( x \), in meters, can be approximated by the following function. The function is valid for values of \( x \) between 0 and 10,000.

\[ f(x) = 1038(1.000134)^{x} \]

a. What is the pressure at sea level?

794 millibars

b. The McDonald Observatory in Texas is at an altitude of 2000 meters. What is the approximate atmospheric pressure there?

749 millibars

c. As altitude increases, what happens to atmospheric pressure?

It decreases.
**Rational Exponents**

You have developed the following properties of powers when \( a \) is a positive real number and \( m \) and \( p \) are integers.

\[
a^m \cdot a^n = a^{m+n} \quad \quad \quad \quad \quad (ab)^n = a^nb^n \quad \quad \quad \quad \quad a^m = 1
\]

\[
(a^m)^n = a^{mn} \quad \quad \quad \quad \quad a^0 = 1 \quad \quad \quad \quad \quad a^{-m} = \frac{1}{a^m}
\]

Exponents need not be restricted to integers. We can define rational exponents so that operations involving them will be governed by the properties for integer exponents.

\[
\frac{a^m}{a^n} = a^{m-n} \quad \quad \quad \quad \quad \quad \quad \quad \quad (a^m)^{1/n} = a^{m/n}
\]

Example 1  Write \( \sqrt{a^2} \) in exponential form.

\[
\sqrt{a^2} = a^{1/2}
\]

Example 2  Write \( a^{3/2} \) in radical form.

\[
a^{3/2} = \sqrt{a^3}
\]

Example 3  Evaluate \( a^{3/2} \) and \( a^{-1/2} \) for \( a = 9 \).

\[
a^{3/2} = 9^{3/2} = 27 \quad \quad \quad \quad \quad \quad \quad \quad \quad a^{-1/2} = 9^{-1/2} = \frac{1}{3}
\]

Write each expression in radical form.

1. \( b^{1/2} \sqrt{b^{3/5}} \)
2. \( 3^{1/2} \sqrt{3} \)
3. \( (3a)^{1/2} \sqrt{3c} \)

Write each expression in exponential form.

4. \( \sqrt{b^3} \quad b^{3/2} \)
5. \( \sqrt{a} \quad (3a^2)^{1/2} \quad 2a^{1/2} \)
6. \( 2 \cdot \sqrt{b} \quad 2b^{1/2} \)

Perform the operation indicated. Answers should show positive exponents only.

7. \( (a/b)^{1/2} \quad a^{1/2} \)
8. \( \frac{-8a^{1/2}}{2a} \quad 4a^{1/2} \)
9. \( \frac{b^{1/2}}{b} \quad b^{1/2} \)
10. \( \sqrt{a} \quad \sqrt{b} \quad a \sqrt{a} \quad b \sqrt{b} \)
11. \( (a^2 b^{1/2})^{1/2} \quad \frac{b^{1/2}}{a} \)
12. \( -2a^{1/2}b^{3/2} \quad (5b^{1/2})^{1/2} \)
13. \( -10a^{1/2} \quad b^{1/2} \)

**Example 2**

Find \( a^{3/2} \) and \( a^{-1/2} \) for \( a = 81 \).

\[
a^{3/2} = 81^{3/2} = 27 \quad \quad \quad \quad \quad \quad \quad \quad \quad a^{-1/2} = 81^{-1/2} = \frac{1}{9}
\]

Write each expression in radical form.

1. \( 3 \cdot \sqrt{b} \)
2. \( \sqrt{3} \)
3. \( \sqrt{3c} \)

Write each expression in exponential form.

4. \( b^{3/2} \)
5. \( (3a^2)^{1/2} \)
6. \( 2 \cdot \sqrt{b} \)

Perform the operation indicated. Answers should show positive exponents only.

7. \( (a/b)^{1/2} \)
8. \( \frac{-8a^{1/2}}{2a} \)
9. \( \frac{b^{1/2}}{b} \)
10. \( \sqrt{a} \quad \sqrt{b} \quad a \sqrt{a} \quad b \sqrt{b} \)
11. \( (a^2 b^{1/2})^{1/2} \)
12. \( -2a^{1/2}b^{3/2} \)
13. \( -10a^{1/2} \)

**Example 3**

Find \( a^{3/2} \) and \( a^{-1/2} \) for \( a = 81 \).

\[
a^{3/2} = 81^{3/2} = 27 \quad \quad \quad \quad \quad \quad \quad \quad \quad a^{-1/2} = 81^{-1/2} = \frac{1}{9}
\]

Write each expression in radical form.

1. \( 3 \cdot \sqrt{b} \)
2. \( \sqrt{3} \)
3. \( \sqrt{3c} \)

Write each expression in exponential form.

4. \( b^{3/2} \)
5. \( (3a^2)^{1/2} \)
6. \( 2 \cdot \sqrt{b} \)

Perform the operation indicated. Answers should show positive exponents only.

7. \( (a/b)^{1/2} \)
8. \( \frac{-8a^{1/2}}{2a} \)
9. \( \frac{b^{1/2}}{b} \)
10. \( \sqrt{a} \quad \sqrt{b} \quad a \sqrt{a} \quad b \sqrt{b} \)
11. \( (a^2 b^{1/2})^{1/2} \)
12. \( -2a^{1/2}b^{3/2} \)
13. \( -10a^{1/2} \)

**Example 1**

Write an equation to represent the population of Johnson City since 2000.

In 2010, the population of Johnson City will be about 34,256.

**Example 2**

The population of Johnson City in 2000 was 25,000. Since then, the population has grown at an average rate of 3.2% each year.

a. Write an equation to represent the population of Johnson City since 2000.

The rate 3.2% can be written as 0.032.

\[
y = 25,000(1.032)^t
\]

b. According to the equation, what will the population of Johnson City be in the year 2010?

In 2010, \( t = 10 \) days.

\[
y = 25,000(1.032)^{10} = 34,256
\]

In 2010, the population of Johnson City will be about 34,256.

**Example 3**

Investment The Garcias have $12,000 in a savings account. The bank pays 3.5% interest on savings accounts, compounded monthly.

Find the balance in 3 years.

The rate 3.5% can be written as 0.035.

\[
n = P(1 + \frac{r}{12})^{nt}
\]

\[
A = 12,000(1 + \frac{0.035}{12})^{12} \approx 13,328.09
\]

In three years, the balance of the account will be $13,328.09.

**Exercises**

1. Population

The population of the United States has been increasing at an average annual rate of 0.91%. If the population of the United States was about 303,146,000 in the year 2008, predict the U.S. population in the year 2022. About 308,385,845

2. Investment

Determine the amount of an investment of $25,000 if it is invested at an interest rate of 5.2% compounded quarterly for 12 years.

$185,888.87

3. Population

It is estimated that the population of the world is increasing at an average annual rate of 1.3%. If the population of the world was about 6,460,000,000 in the year 2008, predict the world population in the year 2015. About 7,084,881,769

4. Investment

Determine the amount of an investment of $100,000 if it is invested at an interest rate of 6.25% compounded quarterly for 10 years.
9-7 Study Guide and Intervention (continued)

Growth and Decay

Exponential Decay Radioactive decay and depreciation are examples of exponential decay. This means that an initial amount decreases at a steady rate over a period of time.

Example

DEPRECIATION The original price of a tractor was $45,000. The value of the tractor decreases at a steady rate of 12% per year.

a. Write an equation to represent the value of the tractor since it was purchased.

The rate 12% can be written as 0.12.

\[
y = a(1 - r)^t
\]

General equation for exponential decay

\[
y = 45,000(1 - 0.12)^t
\]

Simplify.

\[
y = 45,000(0.88)^t
\]

b. What is the value of the tractor in 5 years?

\[
y = 45,000(0.88)^5
\]

Use a calculator.

\[
y \approx 23,747.94
\]

In 5 years, the tractor will be worth about $23,747.94.

Exercises

1. POPULATION The population of Bulgaria has been decreasing at an annual rate of 0.89%. If the population of Bulgaria was about 7,450,349 in the year 2000, predict its population in the year 2015.

2. DEPRECIATION Mr. Gossell is a machinist. He bought some new machinery for about $125,000. He wants to calculate the value of the machinery over the next 10 years for tax purposes. If the machinery depreciates at the rate of 15% per year, what is the value of the machinery (to the nearest $100) at the end of 10 years?

3. ARCHAEOLOGY The half-life of a radioactive element is defined as the time that it takes for one-half a quantity of the element to decay. Radioactive carbon-14 is found in all living organisms and has a half-life of 5730 years. Consider a living organism with an original concentration of carbon-14 of 100 grams.

   a. If the organism lived 5730 years ago, what is the concentration of carbon-14 today?

   b. If the organism lived 11,460 years ago, determine the concentration of carbon-14 today.

4. DEPRECIATION A new car costs $32,000. It is expected to depreciate 12% each year for 4 years and then depreciate 8% each year thereafter. Find the value of the car in 6 years.

5. MANUFACTURING Zeller Industries bought a piece of weaving equipment for $60,000. It is expected to depreciate at the rate of 10% per year. Find the value of the equipment after 6 years.

6. HOUSING Mr. and Mrs. Boyce bought a house for $96,000 in 1995. The real estate broker indicated that houses in their area were appreciating at an average annual rate of 7%. If the appreciation remained steady at this rate, what was the value of the Boyce's home in 2009?
1. COMMUNICATIONS  The value of a new sports radio station is depreciated by about 14.3% per year.
   a. Write an equation for the number of sports radio stations for $t$ years after 1996.
      $R = 220(1.143)^t$
   b. If the trend continues, predict the number of sports radio stations in 2010.
      About 1429 stations

2. INVESTMENTS  Determine the amount of an investment if $500 is invested at an interest rate of 4.25% compounded quarterly for 12 years.
   $830.41$

3. INVESTMENTS  Determine the amount of an investment if $300 is invested at an interest rate of 6.75% compounded semiannually for 20 years.
   $1131.73$

4. HOUSING  The Greens bought a condominium for $110,000 in 2005. If its value appreciates at an average rate of 6% per year, what will the value be in 2010?
   About $147,205$

5. DEFORESTATION  During the 1990s, the forested area of Guatemala decreased at an average rate of 1.7%.
   a. If the forested area in Guatemala in 1990 was about 34,400 square kilometers, write an equation for the forested area for $t$ years after 1990.
      $C = 34,400(0.983)^t$
   b. If this trend continues, predict the forested area in 2015.
      About 22,407.65 km$^2$

6. BUSINESS  A piece of machinery valued at $15,000 depreciates at a steady rate of 10% per year. What will be the value of the piece of machinery be after 7 years?
   $6182$

7. TRANSPORTATION  A new car costs $18,000. It is expected to depreciate at an average rate of 12% per year. Find the value of the car 8 years after purchase.
   About $6473$

8. POPULATION  In 2007 the U.S. Census Bureau estimated the population of the United States was about 301 million. If the growth rate of the population increases at about 0.69% per year, what will the population be in 2020?
   About 340 million
### 9-7 Enrichment

**Continuously Compounding Interest**

You can use the formula for compound interest $A = P\left(1 + \frac{r}{n}\right)^{nt}$ when $n$, the number of times a year interest is compounded, is known. A special type of compound interest calculation regularly used in finance is continuously compounded interest, where $n$ approaches infinity.

<table>
<thead>
<tr>
<th>$\text{Time (Years)}$</th>
<th>$\text{Interest Rate}$</th>
<th>$\text{Final Value}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5%</td>
<td>$1051.16$</td>
</tr>
<tr>
<td>2</td>
<td>8%</td>
<td>$1104.94$</td>
</tr>
<tr>
<td>3</td>
<td>10%</td>
<td>$1161.47$</td>
</tr>
<tr>
<td>4</td>
<td>10%</td>
<td>$1220.90$</td>
</tr>
<tr>
<td>5</td>
<td>10%</td>
<td>$1283.36$</td>
</tr>
<tr>
<td>6</td>
<td>10%</td>
<td>$1349.02$</td>
</tr>
<tr>
<td>7</td>
<td>10%</td>
<td>$1418.04$</td>
</tr>
<tr>
<td>8</td>
<td>10%</td>
<td>$1490.59$</td>
</tr>
<tr>
<td>9</td>
<td>10%</td>
<td>$1566.85$</td>
</tr>
<tr>
<td>10</td>
<td>10%</td>
<td>$1647.01$</td>
</tr>
</tbody>
</table>

**Example**

**INVESTING**

Mr. Rivera placed $5000 in an investment account that has an interest rate of 9.5% per year.

a. How much money will be in the account after 5 years if interest is compounded monthly?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 5000\left(1 + \frac{0.095}{12}\right)^{12(5)}$

$A = 5000(1.0079)^{60}$

$A = 8025.05$

There will be about $8025.05 in the account if interest is compounded monthly.

b. How much money will be in the account after 5 years if interest is compounded continuously?

$A = Pe^{rt}$

$P = 5000, \ e = 2.71828,$

$r = 0.095,$ and

$t = 5$

$A = 5000 \times 2.71828^{0.095 \times 5}$

$A = 8040.07$

There will be about $8040.07 in the account if interest is compounded continuously.

### Exercises

1. **SAVINGS**

   Mr. Harris saves $20,000 in a money-market account at an interest rate of 5.2%.

   a. Determine the value of his investment after 10 years if interest is compounded quarterly.

   $P = 20,000, \ r = 0.052, \ n = 4, \ t = 10$

   $A = 20000(1.013)^{40}$

   $A = 33,528.01$

   The account will have a value of $33,528.01.

   b. Determine the value of his investment after 10 years if interest is compounded continuously.

   $A = Pe^{rt}$

   $P = 20000, \ e = 2.71828, \ r = 0.052, \ t = 10$

   $A = 20000 \times 2.71828^{0.052 \times 10}$

   $A = 33,640.54$

2. **COLLEGE SAVINGS**

   Shannon is choosing between two different savings accounts to keep her college fund in. The first account compounds interest semiannually at a rate of 11.0%. The second account compounds interest continuously at a rate of 10.8%. If Shannon plans to keep her money in the account for 5 years, which account should she choose?

   She should choose the continuous compounding account at 10.8%. After 5 years, the value of the continuously compounding account will be $1.716P$, whereas the quarterly compounding account will only have a value of $1.708P$.

### Spreadsheet Activity

**Compound Interest**

Banks often use spreadsheets to calculate and store financial data. One application is to calculate compound interest on an account.

**Example**

Use a spreadsheet to find the time it will take an investment of $1000 to double. Suppose you can choose from investments that have annual interest rates of 5%, 8%, or 10% compounded monthly.

The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where $P$ is the principal or initial investment, $A$ is the final amount of the investment, $r$ is the annual interest rate, $n$ is the number of times interest is paid, or compounded, each year, and $t$ is the number of years. In this case, $P = 1000$ and $n = 12$.

**Step 1**

Use Column A of the spreadsheet for the years.

**Step 2**

Columns B, C, and D contain the formulas for the final amounts of the investments. Format the cells in these columns as currency so that the amounts are shown in dollars and cents. Each formula will use the values in Column A as the value for $t$. For example, in cell C3, the formula is $1000 * (1 + (0.08/12))^{12 \times 5}$.

Study the spreadsheet for the times when each investment exceeds $2000. At 5%, the $1000 will double in 14 years, at 8% it will double in 9 years, and at 10% it will double in 7 years.

### Exercises

Use the spreadsheet of accounts involving compound interest.

1. How are the doubling times affected if the accounts compound interest quarterly instead of monthly? The 5% and 8% accounts double in the same number of years as before, but the 10% account will double in 8 years.

2. How long will it take each account to reach $4000 if the interest is compounded monthly? quarterly? monthly: 5% in 28 yr, 8% in 18 yr, 10% in 14 yr; quarterly: 5% in 28 yr, 8% in 18 yr, 10% in 15 yr

3. How do the interest rate and the number of times the interest is compounded affect the growth of an investment? The accounts with higher interest rate and interest compounded more often grow more quickly.
Recognize Geometric Sequences

A geometric sequence is a sequence in which each term after the first is found by multiplying the previous term by a nonzero constant \( r \) called the common ratio. The common ratio can be found by dividing any term by its previous term.

Example 1

Determine whether the sequence is arithmetic, geometric, or neither: 21, 63, 189, 567, . . .

Find the ratios of the consecutive terms.
If the ratios are constant, the sequence is geometric.

\[
\text{Ratio: } \frac{63}{21} = \frac{189}{63} = \frac{567}{189} = 3
\]

Because the ratios are constant, the sequence is geometric. The common ratio is 3.

Geometric Sequences as Exponential Functions

The \( n \)th term of a geometric sequence is found by multiplying the previous term by the common ratio \( r \). The formula for the \( n \)th term is:

\[
a_n = a_1 \cdot r^{n-1}
\]

Example 2

Find the next three terms in this geometric sequence:

\(-1215, 405, -135, 45, . . .\)

Step 1 Find the common ratio.

\[
\frac{405}{-1215} = \frac{-135}{45} = -\frac{1}{3}
\]

Step 2 Multiply each term by the common ratio to find the next three terms.

\[
45 \cdot \left(-\frac{1}{3}\right) = -15
\]

\[
-15 \cdot \left(-\frac{1}{3}\right) = 5
\]

\[
5 \cdot \left(-\frac{1}{3}\right) = -\frac{5}{3}
\]

The next three terms of the sequence are \(-15, 5, \) and \(-\frac{5}{3}\).

Exercises

Determine whether each sequence is arithmetic, geometric, or neither. Explain.

1. 1, 2, 4, 8 . . .
   - geometric common ratio = 2;

2. 2, 4, 6, 11 . . .
   - neither common ratio or difference;

3. \frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{12} . . .
   - arithmetic common ratio = \(\frac{1}{2}\);

4. -2, 5, 12, 19 . . .
   - arithmetic common difference = 7;

Find the next three terms in each geometric sequence.

5. 5, 25, 125 . . .
6. 25, 5, 1 . . .
7. \frac{1}{2}, \frac{1}{3}, \frac{1}{4} . . .
8. 32, 256, 2048

Find the next three terms in this geometric sequence:

9. 4, 12, 36 . . .
10. -5, 25, -125 . . .

Find the next three terms in this geometric sequence:

11. 100, 50, 25 . . .
12. -2, 5, 12, 19 . . .

Find the next three terms in this geometric sequence:

13. 320, 80, 20 . . .
14. 54, 18, 6 . . .

Find the next three terms in this geometric sequence:

15. 3, -9, 27 . . .
16. -2, -5, 12 . . .

Find the next three terms in this geometric sequence:

17. 100, 50, 25 . . .
18. 2, 6, 18 . . .

Find the next three terms in this geometric sequence:

19. 4, 12, 36 . . .
20. -5, 25, -125 . . .

Find the next three terms in this geometric sequence:

21. 320, 80, 20 . . .
22. 54, 18, 6 . . .

Find the next three terms in this geometric sequence:

23. 3, -9, 27 . . .
24. -2, -5, 12 . . .
**Skills Practice**

**Geometric Sequences as Exponential Functions**

Determine whether each sequence is arithmetic, geometric, or neither. Explain.

1. 7, 13, 19, 25, ...
   - Arithmetic; common difference is 6.

2. –48, –96, –144, ...
   - Geometric; the common ratio is \(-\frac{3}{2}\).

3. 108, 144, 192, ...
   - Neither; there is no common difference or ratio.

4. \(\frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \ldots\)
   - Geometric; common ratio is \(\frac{3}{2}\).

5. \(-2, 4, -8, 16, \ldots\)
   - Neither; there is no common difference or ratio.

Find the next three terms in each geometric sequence.

6. 1, 3, 9, ...
   - Arithmetic; the common difference is 2.

7. 7, 13, 19, ...
   - Geometric; the common ratio is \(\frac{3}{2}\).

8. 500, 500, 500, ...
   - Arithmetic; common difference is 0.

Find the eighth term of this sequence.

9. \(-4, 8, -16, 32, \ldots\)
   - Geometric; the common ratio is 2.

10. \(1, 3, 9, 27, \ldots\)
    - Arithmetic; common difference is 2.

Write an equation for the \(n\)th term of the geometric sequence 8, 16, 32, ...

11. \(a_1 = 8, r = 2\)
    - \(a_n = 8 \cdot 2^{n-1}\).

12. \(a_1 = 3, r = \frac{3}{2}\)
    - \(a_n = 3 \cdot \left(\frac{3}{2}\right)^{n-1}\).

Write an equation for the \(n\)th term of the geometric sequence 1000, 2000, ...

13. \(a_1 = 1000, r = 2\)
    - \(a_n = 1000 \cdot 2^{n-1}\).

Find the eighth term of this sequence.

14. \(-3, -6, -12, \ldots\)
    - Geometric; the common ratio is 2.

15. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
    - Arithmetic; common difference is \(-\frac{1}{2}\).

16. \(\frac{1}{1000}, \frac{1}{500}, \frac{1}{250}, \ldots\)
    - Geometric; the common ratio is \(-\frac{1}{2}\).

17. \(\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \ldots\)
    - Neither; there is no common difference or ratio.

18. \(\frac{1}{100}, \frac{1}{10}, 1, \ldots\)
    - Arithmetic; common difference is 1.

Find the tenth term of this sequence.

19. \(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{8}, \ldots\)
    - Neither; there is no common difference or ratio.

20. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
    - Geometric; the common ratio is \(\frac{1}{2}\).

21. \(\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\)
    - Arithmetic; common difference is \(-\frac{1}{3}\).

22. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
    - Geometric; the common ratio is \(\frac{1}{2}\).

Write an equation for the \(n\)th term of the geometric sequence 1, 3, 9, ...

23. \(a_1 = 1, r = 3\)
    - \(a_n = 1 \cdot 3^{n-1}\).

Find the seventh term of this sequence.

24. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
    - Geometric; the common ratio is \(\frac{1}{2}\).

25. \(\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\)
    - Arithmetic; common difference is \(-\frac{1}{3}\).

26. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
    - Geometric; the common ratio is \(\frac{1}{2}\).

Write an equation for the \(n\)th term of the geometric sequence 2, 4, 8, ...

27. \(a_1 = 2, r = 2\)
    - \(a_n = 2 \cdot 2^{n-1}\).

Find the seventh term of this sequence.

28. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
    - Geometric; the common ratio is \(\frac{1}{2}\).

29. \(\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\)
    - Arithmetic; common difference is \(-\frac{1}{3}\).

30. \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)
    - Geometric; the common ratio is \(\frac{1}{2}\).
1. **WORLD POPULATION** The CIA estimates the world population is growing at a rate of 1.167% each year. The world population for 2007 was about 6.6 billion.

   a. Write an equation for the world population after \( n \) years. (Hint: The common ratio is not just 0.01167.)
   
   \[ a_n = 6,600,000,000 \cdot 1.01167^n - 1 \]

   b. What will the estimated world population be in 2017?
   
   about 7.4 billion

2. **MUSEUMS** The table shows the annual visitors to a museum in millions. Write an equation for the projected number of visitors after \( n \) years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Visitors (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>13.5</td>
</tr>
<tr>
<td>( n )</td>
<td>( 4 \cdot \left(\frac{3}{2}\right)^{n-1} )</td>
</tr>
</tbody>
</table>

3. **BANKING** Arnold has a bank account with a beginning balance of $5000. He spends one-fifth of the balance each month. How much money will be in the account after 6 months?

   $1310.72

4. **POPULATION** The table shows the projected population of the United States through 2050. Does this table show an arithmetic sequence, a geometric sequence or neither? Explain.

<table>
<thead>
<tr>
<th>Year</th>
<th>Projected Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>282,525,000</td>
</tr>
<tr>
<td>2010</td>
<td>308,356,000</td>
</tr>
<tr>
<td>2020</td>
<td>335,805,000</td>
</tr>
<tr>
<td>2030</td>
<td>363,584,000</td>
</tr>
<tr>
<td>2040</td>
<td>391,946,000</td>
</tr>
<tr>
<td>2050</td>
<td>419,854,000</td>
</tr>
</tbody>
</table>

   Neither, there is no common ratio or difference.

5. **SAVINGS ACCOUNTS** A bank offers a savings account with a 0.5% return each month. Write an equation for the balance of a savings account after \( n \) months. (Hint: The common ratio is not just 0.005.)

   \[ a_n = p \cdot 1.005^n \]

   a. Given an initial deposit of $500, what will the account balance be after 15 months?
   
   $538.84

   b. Write an equation for the balance of a savings account after \( n \) months. (Hint: The common ratio is not just 0.005.)

   \[ a_n = p \cdot 1.005^n \]

   c. Given an initial deposit of $500, what will the account balance be after 15 months?

   $538.84

   d. Graph the data you found in the chart as ordered pairs and connect with a smooth curve.

   1. Graph the data you found in the chart as ordered pairs and connect with a smooth curve.

   2. What type of function is your graph from problem 1? Write an equation that can be used to determine the number of good deeds on any given day, \( x \).

   3. How many good deeds will be performed on Day 21? 10,460,353,203

   4. Use this formula to determine the approximate number of good deeds that have been performed through Day 21.

   5. Look up the world population. How does your number from Exercise 4 compare to the world population?

   The world population is approximately 6,560,526,046. The number of good deeds performed is greater than the entire world population.
**9-9 Study Guide and Intervention**

**Analyzing Functions with Successive Differences and Ratios**

**Identify Functions**
Linear functions, quadratic functions, and exponential functions can all be used to model data. The general forms of the equations are listed below.

- **Linear Function** \( y = mx + b \)
- **Quadratic Function** \( y = ax^2 + bx + c \)
- **Exponential Function** \( y = ab^x \)

You can also identify data as linear, quadratic, or exponential based on patterns of behavior of their \( y \)-values.

**Example 1**
Graph the set of ordered pairs \((-3, 2), (-2, -1), (-1, -2), (0, -1), (1, 2)) Determine whether the ordered pairs represent a **linear function**, a **quadratic function**, or an **exponential function**.

The ordered pairs appear to represent a quadratic function.

**Example 2**
Look for a pattern in the table to determine which model best describes the data.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0.5</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Start by comparing the first differences. The first differences are not all equal. The table does not represent a linear function.

Find the second differences and compare. The second differences are equal. Therefore, the table represents a quadratic function.

The ratios of successive \( y \)-values are equal. Therefore, the table can be modeled by an exponential function.

The equation for exponential function:

\[ y = 3 \cdot 2^x \]

**Exercises**
Graph each set of ordered pairs. Determine whether the ordered pairs represent a **linear function**, a **quadratic function**, or an **exponential function**.

1. \((0, -1), (1, 1), (2, 3), (3, 5)\) **Linear**
2. \((-3, -1), (-2, -4), (-1, -5), (0, -4), (1, -1)\) **Quadratic**
3. \((x, y) = (-2, -1), (0, 1), (1, 2)\) **Linear**
4. \((x, y) = (-2, -1), (0, 1), (1, 2)\) **Exponential**

Look for a pattern in each table to determine which model best describes the data.

3. \(x\) \(-2\) \(-1\) \(0\) \(1\) \(2\)
   \(y\) \(6\) \(3\) \(4\) \(3\) \(2\) **Linear**
4. \(x\) \(-2\) \(-1\) \(0\) \(1\) \(2\)
   \(y\) \(6\) \(2\) \(1\) \(0.5\) \(0.25\) **Exponential**

**Write Equations**
Once you find the model that best describes the data, you can write an equation for the function.

**Basic Forms**
- **Linear Function** \( y = mx + b \)
- **Quadratic Function** \( y = ax^2 + bx + c \)
- **Exponential Function** \( y = ab^x \)

**Step 1**
Determine whether the data is modeled by a linear, quadratic, or exponential function.

- First differences: \(4, 2, 1, 0.5, 0.25\)
- Second differences: \(2, 0.5, 0.25\)
- \(y\)-value ratios: \(3, 2, 1, 0.5, 0.25\)

**Step 2**
Write an equation for the function that models the data.

- **Linear** \( y = 3x + 2 \)
- **Quadratic** \( y = 3x^2 + 1 \)
- **Exponential** \( y = 3 \cdot 2^x \)

An equation that models the data is \( y = 3.2^x \). To check the results, you can verify that the other ordered pairs satisfy the function.

**Exercises**
Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

1. \((x, y) = (-2, 1), (0, 3), (1, 6)\) **Quadratic** \( y = 3x^2 \)
2. \((x, y) = (-2, 1), (0, 3), (1, 7)\) **Linear** \( y = 3x + 1 \)
3. \((x, y) = (-2, 1), (0, 3), (1, 7)\) **Exponential** \( y = 3 \cdot 2^x \)
Graph each set of ordered pairs. Determine whether the ordered pairs represent a linear function, a quadratic function, or an exponential function.

1. (2, 3), (1, 1), (0, –1), (–1, –3), (–3, –5)

2. (–1, 0.5), (0, 1), (1, 2), (2, 4)

3. (–2, 4), (–1, 1), (0, 0), (1, 1), (2, 4)

4. (–3, 5), (–2, 2), (–1, 1), (0, 2), (1, 5)

Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

5. \(x\) \(-3\) \(-1\) 0 1 2 3 5 \(y\) \(-5\) –2 1 4 7
   - linear; \(y = \frac{3}{2}x - \frac{1}{2}\)
   - exponential; \(y = 9 \cdot (0.5)^x\)

6. \(x\) \(-2\) –1 0 1 2 \(y\) 0.02 0.2 0.5 2 20 200
   - exponential; \(y = 2 \cdot 10^x\)

7. \(x\) \(-1\) 0 1 2 3 \(y\) 6 12 36 108
   - quadratic; \(y = 6x^2\)

8. \(x\) \(-2\) –1 0 1 2 \(y\) 0 8 12 8 0
   - linear; \(y = -9x\)

9. \(x\) \(-2\) –1 0 1 2 3 4 5 \(y\) 8 4 0 4 8
   - linear; \(y = 4x\)

---

**Skills Practice**

Graph each set of ordered pairs. Determine whether the ordered pairs represent a linear function, a quadratic function, or an exponential function.

1. (4, 0.5), (3, 1.5), (2, 2.5), (1, 3.5), (0, 4.5)

2. (–1, 0.5), (0, 1), (1, 2), (2, 4)

3. (–2, 4), (–1, 1), (0, 0), (1, 1), (2, 4)

4. (–3, 5), (–2, 2), (–1, 1), (0, 2), (1, 5)

---

**Practice**

Graph each set of ordered pairs. Determine whether the ordered pairs represent a linear function, a quadratic function, or an exponential function.

1. (2, 3), (1, 1), (0, –1), (–1, –3), (–3, –5)

2. (–1, 0.5), (0, 1), (1, 2), (2, 4)

3. (–2, 4), (–1, 1), (0, 0), (1, 1), (2, 4)

4. (–3, 5), (–2, 2), (–1, 1), (0, 2), (1, 5)

Look for a pattern in each table of values to determine which model best describes the data. Then write an equation for the function that models the data.

5. \(x\) \(-3\) –1 1 3 5 \(y\) \(-5\) –2 1 4 7
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---

**INSECTS**

The local zoo keeps track of the number of dragonflies breeding in their insect exhibit each day.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dragons</td>
<td>9</td>
<td>18</td>
<td>36</td>
<td>72</td>
<td>144</td>
</tr>
</tbody>
</table>

a. Determine which function best models the data. exponential
b. Write an equation for the function that models the data. \(y = 9 \cdot 2^x\)
c. Use your equation to determine the number of dragonflies that will be breeding after 9 days. 4608 dragonflies
**Chapter 9**

**NAME**

**DATE**

**PERIOD**

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**NAME**

**DATE**

**PERIOD**

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**Chapter 9**

**NAME**

**DATE**

**PERIOD**

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**Chapter 9**

**NAME**

**DATE**

**PERIOD**

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**Word Problem Practice**

1. **WEATHER**

   The San Mateo weather station records the amount of rainfall since the beginning of a thunderstorm. Data for a storm is recorded as a series of ordered pairs shown below, where the x value is the time in minutes since the start of the storm, and the y value is the amount of rain in inches that has fallen since the start of the storm.

   (2, 0.3), (4, 0.6), (6, 0.9), (8, 1.2), (10, 1.5)  Graph the ordered pairs. Determine whether the ordered pairs represent a linear function, a quadratic function, or an exponential function.

2. **NUCLEAR WASTE**

   Radioactive material slowly decays over time. The amount of time needed for an amount of radioactive material to decay to half its initial quantity is known as its half-life. Consider a 20-gram sample of a radioactive isotope.

   a. Is radioactive decay a linear decay, quadratic decay, or an exponential decay?

   b. Write an equation to determine how many grams of the isotope will remain after n half-lives.

   c. How many grams of the isotope will remain after 11 half-lives?

   d. Plutonium-238 is one of the most dangerous waste products of nuclear power plants. If the half-life of plutonium-238 is 70,176 years, how long would it take for a 20 gram sample of plutonium-238 to decay to 0.078 grams?

---

**Answers** (Lesson 9-9)

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**Word Problem Practice**

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OFFENDING COMMAND:

STACK: