## 7 Anticipation Guide
### Polynomials

**Step 1** Before you begin Chapter 7

- Read each statement.
- Decide whether you Agree (A) or Disagree (D) with the statement.
- Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

<table>
<thead>
<tr>
<th>STEP 1</th>
<th>Statement</th>
<th>STEP 2 A, D, or NS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, D, or NS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. When multiplying two powers that have the same base, multiply the exponents.</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>2. ( (4^3)^2 ) is equivalent to ( 4^{6} ).</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>3. To divide two powers that have the same base, subtract the exponents.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>4. ( (\frac{2}{3})^3 ) is the same as ( \frac{8}{27} ).</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>5. A polynomial may contain one or more monomials.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>6. The sum of the two polynomials ( 3x^3y^5 - 5y^5 + 8x^3 ) is 3 because the highest exponent is 3.</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>7. The product of two polynomials ( (3xy - 4x^2 + 2y) ) and ( (6x^2 + 2x y - 7) ) in simplest form is ( 5x^3y + 2x^2 + 2y^2 - 7 ).</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>8. ( 4m^2 + 2m - 31 - 9m^2 - m + 31 ) is equal to ( 3m^2 + m ).</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>9. Because there are different exponents in each factor, the distributive property cannot be used to multiply ( 3n^2 ) by ( (2x^2 + 4n - 10) ).</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>10. The FOIL method of multiplying two binomials stands for <strong>First, Outer, Inner, Last</strong>.</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>11. The square of ( r + t ) or ( r^2 + t^2 ), will always equal ( r^2 + t^2 ).</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>12. The product of ( (x + y) ) and ( (x - y) ) will always equal ( x^2 - y^2 ).</td>
<td>A</td>
<td></td>
</tr>
</tbody>
</table>

**Step 2** After you complete Chapter 7

- Reread each statement and complete the last column by entering an A or a D.
- Did any of your opinions about the statements change from the first column?
- For those statements that you mark with a D, use a piece of paper to write an example of why you disagree.
7-1 Study Guide and Intervention (continued)

Multiplying Monomials

Simplify Expressions. An expression of the form \( (x^m)^n \) is called a power of a power and represents the product you obtain when \( x^n \) is used as a factor \( n \) times. To find the power of a power, multiply exponents.

<table>
<thead>
<tr>
<th>Power of a Power</th>
<th>For any number ( a ) and all integers ( m ) and ( n ), ( (a^n)^m = a^{mn} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power of a Product</td>
<td>For any number ( a ) and all integers ( m ) and ( n ), ( (a^n)(b^m) = a^{n+m}b^{m+n} ).</td>
</tr>
</tbody>
</table>

We can combine and use these properties to simplify expressions involving monomials.

Example

Simplify \( (-2a^2b)^3 \).

\[
(-2a^2b)^3 = (-2)^3(a^2)^3b^3 \\
= -8a^6b^3 \\
\]

The product is \(-8a^6b^3\).

Exercises

Simplify each expression.

1. \( (5x^3)^2 \)  
2. \( (a^n)^4 \)  
3. \( (x^2yz)^3 \)  
4. \(-3ab^{12} \)  
5. \(-3ab^9 \)  
6. \( (4x^3y^2)^2 \)  
7. \( (6a^3b)^2 \)  
8. \( (4x^2y^3)^2 \)  
9. \( (x^2y^3)^2 \)  
10. \( 2a^3b^2 \)  
11. \( (5x^2y^3)^2 \)  
12. \( (-3a^3b^2)^2 \)  
13. \( (2a^3b^2)^2 \)  
14. \( (2x^2y^3)^2 \)  
15. \(-8x^4y^5 \)  
16. \( -6a^2y^3z^2 \)  
17. \( -3a^2y^3z^2 \)  
18. \(-3a^2y^3z^2 \)  
19. \(-3a^2y^3z^2 \)  
20. \(-3a^2y^3z^2 \)  
21. \(-3a^2y^3z^2 \)  
22. \(-3a^2y^3z^2 \)  
23. \(-3a^2y^3z^2 \)  
24. \(-3a^2y^3z^2 \)  
25. \( (2x^3y^2)^2 \)  
26. \( (4x^2y^3)^2 \)  
27. \( (2x^3y^2)^2 \)  

GEOMETRY Express the area of each figure as a monomial.

- \( x^2 \)  
- \( c^2d^2 \)  
- \( 18p^5 \)  

Chapter 7
4. \text{SPORTS} The volume of a sphere is given by the formula \(V = \frac{4}{3}\pi r^3\), where \(r\) is the radius of the sphere. Find the volume of air in three different basketballs. Use \(\pi \approx 3.14\). Round your answers to the nearest whole number.

<table>
<thead>
<tr>
<th>Ball</th>
<th>Radius (in.)</th>
<th>Volume (in(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child's</td>
<td>4</td>
<td>268</td>
</tr>
<tr>
<td>Woman's</td>
<td>4.5</td>
<td>382</td>
</tr>
<tr>
<td>HTH</td>
<td>4.8</td>
<td>463</td>
</tr>
</tbody>
</table>

Source: WikiAnswers

5. \text{ELECTRICITY} An electrician uses the formula \(W = FR\), where \(W\) is the power in watts, \(F\) is the current in amperes, and \(R\) is the resistance in ohms.

a. Find the power in a household circuit that has 20 amperes of current and 5 ohms of resistance. \(2000\) watts

b. If the current is reduced by one half, what happens to the power? The power is one-fourth the previous amount.

6. \text{CIVIL ENGINEERING} A developer is planning a sidewalk for a new development. The sidewalk can be installed in rectangular sections that have a fixed width of 3 feet and a length that can vary. Assuming that each section is the same length, express the area of a 4-section sidewalk as a monomial. \(12x\)

7. \text{GEOMETRY} Express the area of each figure as a monomial.

\[
\begin{align*}
15. & \quad 18a^3b^5 \\
16. & \quad (25x^4)\pi \\
17. & \quad 12a^3b^3
\end{align*}
\]

8. \text{GEOMETRY} Express the volume of each solid as a monomial.

\[
\begin{align*}
18. & \quad 27h^6 \\
19. & \quad m^4n^3 \\
20. & \quad (63g^3)\pi
\end{align*}
\]

9. \text{COUNTING} A panel of four light switches can be set in 2\(^4\) ways. A panel of five light switches can set in twice this many ways. In how many ways can five light switches be set? 2\(^5\) or 32

10. \text{HOBBIES} Tawa wants to increase her rock collection by a power of three this year and then increase it again by a power of two next year. If she has 2 rocks now, how many rocks will she have after the second year? 2\(^5\) or 32
7-1 Enrichment

An Wang

An Wang (1920–1990) was an Asian-American who became one of the pioneers of the computer industry in the United States. He grew up in Shanghai, China, but came to the United States to further his studies in science. In 1948, he invented a magnetic pulse controlling device that vastly increased the storage capacity of computers. He later founded his own company, Wang Laboratories, and became a leader in the development of desktop calculators and word processing systems. In 1988, Wang was elected to the National Inventors Hall of Fame.

Binary numbers. To find the decimal value of a binary number, you use the digits to write a polynomial in 2. For instance, this is how to find the decimal value of the number $1001101_2$.

$1001101_2 = 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 64 + 0 + 0 + 8 + 4 + 0 + 1$

$= 77$

Find the decimal value of each binary number.

1. $1111_2$
2. $10000_2$
3. $1000011_2$
4. $1011001_2$

Write each decimal number as a binary number.

5. $15_{10}$
6. $16_{10}$
7. $195_{10}$
8. $185_{10}$

9. The chart at the right shows a set of decimal code numbers that is used widely in storing letters of the alphabet in a computer's memory. Find the code numbers for the letters of your name. Then write the code for your name using binary numbers. Answers will vary.

The American Standard Code for Information Interchange (ASCII)

<table>
<thead>
<tr>
<th>Letter</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>65</td>
</tr>
<tr>
<td>B</td>
<td>66</td>
</tr>
<tr>
<td>C</td>
<td>67</td>
</tr>
<tr>
<td>D</td>
<td>68</td>
</tr>
<tr>
<td>E</td>
<td>69</td>
</tr>
<tr>
<td>F</td>
<td>70</td>
</tr>
<tr>
<td>G</td>
<td>71</td>
</tr>
<tr>
<td>H</td>
<td>72</td>
</tr>
<tr>
<td>I</td>
<td>73</td>
</tr>
<tr>
<td>J</td>
<td>74</td>
</tr>
<tr>
<td>K</td>
<td>75</td>
</tr>
<tr>
<td>L</td>
<td>76</td>
</tr>
<tr>
<td>M</td>
<td>77</td>
</tr>
<tr>
<td>N</td>
<td>78</td>
</tr>
<tr>
<td>O</td>
<td>79</td>
</tr>
<tr>
<td>P</td>
<td>80</td>
</tr>
<tr>
<td>Q</td>
<td>81</td>
</tr>
<tr>
<td>R</td>
<td>82</td>
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<td>S</td>
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<td>T</td>
<td>84</td>
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<tr>
<td>U</td>
<td>85</td>
</tr>
<tr>
<td>V</td>
<td>86</td>
</tr>
<tr>
<td>W</td>
<td>87</td>
</tr>
<tr>
<td>X</td>
<td>88</td>
</tr>
<tr>
<td>Y</td>
<td>89</td>
</tr>
<tr>
<td>Z</td>
<td>90</td>
</tr>
</tbody>
</table>

10. $5^5$ or $125$
11. $2^5$
12. $3^3$
13. $p^3q^3$
14. $1000011_2$
15. $1011001_2$
16. $10111001_2$
17. $1110101_2$
18. $1100011_2$
19. $10000_2$
20. $1111_2$
21. $1011001_2$
22. $10111001_2$
Simplify each expression. Assume that no denominator equals zero.

1. \( \frac{6^5}{6^7} \) or 6
2. \( \frac{9^3}{9^4} \) or 6561
3. \( \frac{x^2}{x^3} \)
4. \( \frac{-b^1}{b^4} \) or \( \frac{1}{b^3} \)
5. \( \frac{m^3}{m^1} \)
6. \( \frac{8^1}{5^2} \) or 3d
7. \( \frac{3^{n^1}}{4^{n^2}} \) or \( \frac{3}{64n} \)
8. \( \frac{m^4}{x^5} \) or \( \frac{x^2}{w^3} \)
9. \( \frac{a^{h^1}}{a^{b^9}} \) or \( \frac{a^{b^9}}{a^{h^1}} \)
10. \( \frac{m^p^0}{m^p^0} \) or \( m^p^0 \)
11. \( \frac{-2^{m^1}}{7^{m^2}} \) or \( \frac{3w}{x^3} \)
12. \( \frac{3^{m^1}}{5^{n^2}} \) or \( \frac{-8x^2y^z}{5} \)
13. \( \frac{4^{m^1}}{7^{n^1}} \) or \( \frac{16p}{49r^4} \)
14. \( \frac{-1}{4^2} \) or \( \frac{1}{256} \)
15. \( \frac{8^8}{1^8} \) or \( \frac{1}{64} \)
16. \( \frac{9}{5^3} \) or \( \frac{9}{25} \)
17. \( \frac{9}{1^1} \) or \( \frac{11}{9} \)
18. \( \frac{h^1}{h^2} \) or \( h^\frac{1}{2} \)
19. \( k^{(h)}(k^{(h)})^{-\frac{1}{k}} \)
20. \( k^{(h)}(k^{(h)})^{-\frac{m^3}{kt}} \)
21. \( \frac{f^1}{f^3} \) or \( \frac{1}{f^2} \)
22. \( \frac{16p^2}{2p^2} \) or \( 16p^2 \)
23. \( \frac{g^1}{h^1} \) or \( \frac{g}{h} \)
24. \( \frac{15x^y}{5^0} \) or \( 3x^y \)
25. \( \frac{-15x^y}{5^0} \) or \( \frac{3}{u^4} \)
26. \( \frac{48x^y}{6y^2} \) or \( \frac{8x^y}{2} \)
### Chapter 7

#### Practice

**Dividing Monomials**

Simplify each expression. Assume that no denominator equals zero.

1. \( \frac{8^8}{6} \) or 4096
2. \( \frac{a^xy^z}{a^y} \)
3. \( \frac{x^y}{y} \)
4. \( \frac{3y^2}{4} \)
5. \( \frac{5d^2}{4} - \frac{5d^2}{2} \)
6. \( \frac{2yz}{3} \)
7. \( \frac{64p^q}{27h^3} \)
8. \( \frac{36w^m}{49p^2y^2} \)
9. \( \frac{1}{2} \) \( \frac{1}{6x^2} \)
10. \( \frac{1}{x^2-y^2} \)
11. \( \frac{p}{q} \) \( \frac{1}{x^2+y^2} \)
12. \( \frac{1}{2} \) \( \frac{1}{144} \)
13. \( \frac{1}{3} \) \( \frac{1}{4} \)
14. \( \frac{1}{2} \) \( \frac{1}{5} \)
15. \( \frac{1}{2} \) \( \frac{1}{7} \)
16. \( \frac{3}{u^2} \)
17. \( \frac{2c^f}{1} \)
18. \( \frac{1}{4} \)
19. \( \frac{3}{5} \) \( \frac{3}{4} \)
20. \( \frac{1}{2} \) \( \frac{1}{3} \)
21. \( \frac{1}{2} \) \( \frac{1}{2} \)
22. \( \frac{m^2}{n^2} \)
23. \( \frac{1}{j^2} \)
24. \( \frac{1}{2} \) \( \frac{1}{4} \)
25. \( \frac{q^p}{q^m} \)
26. \( \frac{1}{7d^2} \)
27. \( \frac{2y^2}{4y^2} \)

#### Word Problem Practice

**Dividing Monomials**

1. **CHEMISTRY**
   - The nucleus of a certain atom is \( 10^{-11} \) centimeters across. If the nucleus of a different atom is \( 10^{-10} \) centimeters across, how many times as large is it as the first atom? \( 100 \)

2. **SPACE**
   - The Moon is approximately 25 kilimeters away from Earth on average. The Moon stands 25 kilometers high. How many Moon volcanoes, stacked on top of one another, would fit between the surface of the Earth and the Moon? \( 25^2 = 15,625 \)

3. **E-MAIL**
   - Spammers also know as junk e-mails consist of identical messages sent to thousands of e-mail users. People often obtain anti-spam software to filter out the junk e-mails messages they receive. Suppose Yvonne's anti-spam software filtered out 100 e-mails, and she received 1000 e-mails last year. What fraction of her e-mails were filtered out? Write your answer as a monomial. \( 10^{-2} \)

4. **METRIC MEASUREMENT**
   - Consider a dust mote that measures \( 10^{-7} \) millimeters in length and a caterpillar that measures 10 centimeters long. How many times as long as the mote is the caterpillar? \( 10^3 = 100,000 \)

5. **COMPUTERS**
   - In 1995, standard capacity for a personal computer hard drive was 40 megabytes (MB). In 2010, a standard hard drive capacity was 500 gigabytes (GB or Giga). Refer to the table below.

<table>
<thead>
<tr>
<th>Memory Capacity Approximate Conversions</th>
<th>0.001 byte = 1 kilobyte</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^9 bytes = 1 megabyte (meg)</td>
<td>10^10 megabytes = 1 gigabyte (gig)</td>
</tr>
<tr>
<td>10^10 gigabytes = 1 terabyte (ter)</td>
<td>10^11 terabytes = 1 petabyte</td>
</tr>
</tbody>
</table>

   a. The newer hard drives have about how many times the capacity of the 1995 drives? \( 10^2 \)
   b. Predict the hard drive capacity in the year 2025 if this rate of growth continues. \( 6.25 \) petabytes
   c. One kilobyte of memory is what fraction of one terabyte? \( \frac{1}{10^9} = 10^{-9} \)

28. **BIOLOGY**
   - A lab technician draws a sample of blood. A cubic millimeter of the blood contains 22,000 white blood cells and 22,000 red blood cells. What is the ratio of white blood cells to red blood cells? \( \frac{1}{484} \)

29. **COUNTING**
   - The number of three-letter "words" that can be formed with the English alphabet is \( 26^3 \). The number of five-letter "words" that can be formed is \( 26^5 \). How many times more five-letter "words" can be formed than three-letter "words"? \( 676 \)
Patterns with Powers

Use your calculator, if necessary, to complete each pattern.

a. \(2^n = \)

\[\begin{array}{c|c|c|c|c}
\hline
n & 0 & 1 & 2 & 3 \\
\hline
2^n & 1 & 2 & 4 & 8 \\
\hline
\end{array}\]

b. \(5^n = \)

\[\begin{array}{c|c|c|c|c}
\hline
n & 0 & 1 & 2 & 3 \\
\hline
5^n & 1 & 5 & 25 & 125 \\
\hline
\end{array}\]

c. \(4^n = \)

\[\begin{array}{c|c|c|c|c}
\hline
n & 0 & 1 & 2 & 3 \\
\hline
4^n & 1 & 4 & 16 & 64 \\
\hline
\end{array}\]

Study the patterns for \(a, b,\) and \(c\) above. Then answer the questions.

1. Describe the pattern of the exponents from the top of each column to the bottom.

   The exponents decrease by one from each row to the one below.

2. Describe the pattern of the powers from the top of the column to the bottom. To get each power, divide the power on the row above by the base \((2, 5, \text{or} 4)\).

   \[\begin{array}{c|c|c|c|c}
   \hline
   n & 0 & 1 & 2 & 3 \\
   \hline
   \frac{1}{2^n} & 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} \\
   \hline
   \end{array}\]

3. What would you expect the following powers to be?

   \(2^{-1} = \frac{1}{2}, 2^{-2} = \frac{1}{4}\)

4. Refer to Exercise 3. Write a rule. Test it on patterns that you obtain using \(22, 25,\) and \(24\) as bases. Any nonzero number to the zero power equals one.

   \[\begin{array}{c|c|c|c|c}
   \hline
   0^0 & 0^0 & 0^0 & 0^0 \\
   \hline
   0^0 & 0^0 & 0^0 & 0^0 \\
   \hline
   \end{array}\]

   \[\begin{array}{c|c|c|c|c}
   \hline
   0^0 & 0^0 & 0^0 & 0^0 \\
   \hline
   0^0 & 0^0 & 0^0 & 0^0 \\
   \hline
   \end{array}\]

5. Why do \(0^{-1}, 0^{-2},\) and \(0^{-3}\) not exist?

   Negative exponents are not defined unless the base is nonzero.

6. Based upon the pattern, can you determine whether \(0^0\) exists?

   No, since the pattern \(0^0 = 0\) breaks down for \(n \leq 1\).

7. The symbol \(0^0\) is called an indeterminate, which means that it has no unique value. Thus it does not exist as a unique real number. Why do you think that \(0^0\) cannot equal 1?

   Answers will vary. One answer is that if \(0^0 = 1\), then \(1 = \frac{1}{0} = \frac{1^0}{0^0} = (\frac{1}{0})^0\), which is a false result, since division by zero is not allowed. Thus, \(0^0\) cannot equal 1.
**Chapter 7-3 Study Guide and Intervention (continued) Scientific Notation**

**Products and Quotients in Scientific Notation** You can use scientific notation to simplify multiplying and dividing very large and very small numbers.

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluate ((9.2 \times 10^3) / (4 \times 10^9)). Express the result in both scientific notation and standard form.</td>
<td>Evaluate ((6.9 \times 10^2) / (3 \times 10^5)). Express the result in both scientific notation and standard form.</td>
</tr>
<tr>
<td>(\frac{9.2 \times 10^3}{4 \times 10^9})</td>
<td>(\frac{6.9 \times 10^2}{3 \times 10^5})</td>
</tr>
<tr>
<td>Original Expression</td>
<td>Original Expression</td>
</tr>
<tr>
<td>(= \frac{9.2}{4} \times 10^{3-9})</td>
<td>(= \frac{6.9}{3} \times 10^{2-5})</td>
</tr>
<tr>
<td>Commutative and Associative Properties</td>
<td>Product rule for fractions</td>
</tr>
<tr>
<td>(= \frac{9.2}{4} \times 10^{-6})</td>
<td>(= \frac{6.9}{3} \times 10^{-3})</td>
</tr>
<tr>
<td>Product of Powers</td>
<td>Quotient of Powers</td>
</tr>
<tr>
<td>(= 2.3 \times 10^{-6})</td>
<td>(= 0.4 \times 10^{-3})</td>
</tr>
<tr>
<td>(\approx 0.0000023)</td>
<td>(\approx 0.4 \times 10^{-3})</td>
</tr>
<tr>
<td>Standard Form</td>
<td>Standard Form</td>
</tr>
</tbody>
</table>

**Exercises**

Evaluate each quotient. Express the results in both scientific notation and standard form.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(0.4 \times 10^4 / 10^2)</td>
<td>(4 \times 10^2)</td>
</tr>
<tr>
<td></td>
<td>(1.7 \times 10^4 / 170,000,000)</td>
<td>(5.32 \times 10^{-7})</td>
</tr>
<tr>
<td>2.</td>
<td>(6.7 \times 10^3 / (3 \times 10^9))</td>
<td>(0.00201)</td>
</tr>
<tr>
<td></td>
<td>(2.01 \times 10^{-6} / 0.000001)</td>
<td>(1.863 	imes 10^6)</td>
</tr>
<tr>
<td>3.</td>
<td>((1.2 \times 10^{-8}) / (6 \times 10^6))</td>
<td>(5.9 \times 10^{-15})</td>
</tr>
<tr>
<td></td>
<td>(7.2 \times 10^{-10} / 0.0000072)</td>
<td>(4.13 \times 10^{-10})</td>
</tr>
</tbody>
</table>

Evaluate each quotient. Express the results in both scientific notation and standard form.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td>((4.9 \times 10^-4) / (2.5 \times 10^-2))</td>
<td>(2 \times 10^-2)</td>
</tr>
<tr>
<td></td>
<td>(1.96 \times 10^3 / 19.6)</td>
<td>(1.0 \times 10^1)</td>
</tr>
<tr>
<td>8.</td>
<td>((1.6 \times 10^-3) / (4 \times 10^-1))</td>
<td>(4 \times 10^2)</td>
</tr>
<tr>
<td></td>
<td>(4.0 \times 10^3 / 400,000,000)</td>
<td>(5.375 \times 10^{-8})</td>
</tr>
<tr>
<td>9.</td>
<td>((4.2 \times 10^-5) / (6 \times 10^-7))</td>
<td>(7 \times 10^1)</td>
</tr>
<tr>
<td></td>
<td>(7 \times 10^2; 70,000)</td>
<td>(3 \times 10^3; 30)</td>
</tr>
</tbody>
</table>

**Answers**

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(3,400,000,000)</td>
<td>(3.4 \times 10^9)</td>
</tr>
<tr>
<td>2.</td>
<td>(2,091,000)</td>
<td>(2.1 \times 10^6)</td>
</tr>
<tr>
<td>3.</td>
<td>(8.8 \times 10^9)</td>
<td>(8.8 \times 10^9)</td>
</tr>
<tr>
<td>4.</td>
<td>(7.15 \times 10^4)</td>
<td>(7.15 \times 10^4)</td>
</tr>
<tr>
<td>5.</td>
<td>(1.86 \times 10^4)</td>
<td>(1.86 \times 10^4)</td>
</tr>
<tr>
<td>6.</td>
<td>(4.9 \times 10^6)</td>
<td>(4.9 \times 10^6)</td>
</tr>
</tbody>
</table>

Evaluate each product. Express the results in both scientific notation and standard form.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.</td>
<td>((6.1 \times 10^3) \times (2 \times 10^5))</td>
<td>((1.22 \times 10^9))</td>
</tr>
<tr>
<td>14.</td>
<td>((4.4 \times 10^4) \times (1.6 \times 10^2))</td>
<td>((7.04 \times 10^6))</td>
</tr>
<tr>
<td>15.</td>
<td>((8.8 \times 10^9) \times (3.5 \times 10^{-13}))</td>
<td>((3.08 \times 10^{-4}))</td>
</tr>
<tr>
<td>16.</td>
<td>((1.35 \times 10^9) \times (7.2 \times 10^{-5}))</td>
<td>((9.54 \times 10^4))</td>
</tr>
<tr>
<td>17.</td>
<td>((2.2 \times 10^{-5}) \times (8 \times 10^6))</td>
<td>((1.76 \times 10^2))</td>
</tr>
<tr>
<td>18.</td>
<td>((3.4 \times 10^{-4}) \times (5.4 \times 10^6))</td>
<td>((1.87 \times 10^2))</td>
</tr>
</tbody>
</table>

Evaluate each quotient. Express the results in both scientific notation and standard form.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.</td>
<td>((9.2 \times 10^5) / (2 \times 10^5))</td>
<td>((4.6 \times 10^2))</td>
</tr>
<tr>
<td>20.</td>
<td>((4.8 \times 10^5) / (3 \times 10^5))</td>
<td>((1.6 \times 10^2))</td>
</tr>
<tr>
<td>21.</td>
<td>((1 \times 10^6) / (4.3 \times 10^4))</td>
<td>((2.3 \times 10^2))</td>
</tr>
<tr>
<td>22.</td>
<td>((6.8 \times 10^5) / (1.25 \times 10^5))</td>
<td>((5.4 \times 10^0))</td>
</tr>
<tr>
<td>23.</td>
<td>((2.576 \times 10^3) / (7.2 \times 10^5))</td>
<td>((3.6 \times 10^{-2}))</td>
</tr>
<tr>
<td>24.</td>
<td>((8.74 \times 10^3) / (2.376 \times 10^4))</td>
<td>((3.6 \times 10^{-1}))</td>
</tr>
</tbody>
</table>
7-3 Practice
Scientific Notation
Express each number in scientific notation.

1. 1.900,000
2. 1.9 × 10^4
3. 50,040,000,000
4. 5.004 × 10^9

Express each number in standard form.
5. 5.53 × 10^-3
6. 53,000,000
7. 9.13 × 10^-7
8. 9.130

Evaluate each product. Express the results in both scientific notation and standard form.
9. (4.8 × 10^4)(6 × 10^5)
10. (7.5 × 10^4)(3.2 × 10^5)
11. (8.06 × 10^4)(5.5 × 10^-5)
12. (8.1 × 10^4)(1.96 × 10^3)
13. (1.13 × 10^-4)(0.00133)
14. (1.45 × 10^-5)(7.2 × 10^-3)
15. (5.131 × 10^-10)(51,313,000,000,000)
16. (5.131 × 10^-10)(51,313,000,000,000)

Evaluate each quotient. Express the results in both scientific notation and standard form.
17. (4.2 × 10^7) 
18. (9.4 × 10^-4)
19. (7.5 × 10^4) 
20. (7.5 × 10^4)

19. GRAVITATION Isaac Newton’s theory of universal gravitation states that the equation \[ F = \frac{Gm_1m_2}{r^2} \] can be used to calculate the amount of gravitational force in newtons between two point masses \( m_1 \) and \( m_2 \) separated by a distance \( r \). \( G \) is a constant equal to \( 6.67 \times 10^{-11} \) N \( \cdot \) m\(^2\)/kg\(^2\). The mass of the earth \( m_1 \) is equal to 5.97 \times 10^{24} \) kg, the mass of the moon \( m_2 \) is equal to 7.36 \times 10^{22} \) kg, and the distance \( r \) between the two is 384,000,000 m.

a. Express the distance \( r \) in scientific notation. 3.84 \times 10^8 m
b. Compute the amount of gravitational force between the earth and the moon. Express your answer in scientific notation. 1.99 \times 10^{20} newtons

7-3 Word Problem Practice
Scientific Notation

1. PLANETS Neptune’s mean distance from the sun is 4,500,000,000 kilometers. Uranus’ mean distance from the sun is 2,870,000,000 kilometers. Express these distances in scientific notation. Neptune: \( 4.5 \times 10^8 \) km; Uranus: \( 2.87 \times 10^8 \) km

2. PATHOLOGY The common cold is caused by the rhinovirus, which commonly measures \( 2 \times 10^{-8} \) m in diameter. The E. coli bacterium, which causes food poisoning, commonly measures \( 3 \times 10^{-8} \) m in length. Express these measurements in standard form. Rhinovirus: 0.00000002 m; E. coli: 0.00000003 m

3. COMMERCE The 2007 Super Bowl cost $2,600,000,000. A 20-second commercial aired during the 2007 Super Bowl cost $400,000. Express these values in scientific notation. How many times more expensive was it to air an advertisement during the 2007 Super Bowl than the 1967 Super Bowl? 2007 Super Bowl: \( 2.6 \times 10^9 \); 1967 Super Bowl: \( 4.0 \times 10^7 \); 65 \times 10^3 or 65 times more expensive

4. AVOGADRO’S NUMBER Avogadro’s number is an important concept in chemistry. It states that the number 6.022 \times 10^{23} is approximately equal to the number of molecules in 12 grams of carbon 12. Use Avogadro’s number to determine the number of molecules in 5 \times 10^{-3} grams of carbon 12. 2.509 \times 10^{19} molecules

5. COAL RESERVES The table below shows the number of kilograms of coal select countries had in proven reserve at the end of 2006.

<table>
<thead>
<tr>
<th>Country</th>
<th>Coal (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>2.46 \times 10^{12}</td>
</tr>
<tr>
<td>Russia</td>
<td>1.53 \times 10^{12}</td>
</tr>
<tr>
<td>India</td>
<td>0.98 \times 10^{12}</td>
</tr>
<tr>
<td>Romania</td>
<td>4.94 \times 10^{12}</td>
</tr>
</tbody>
</table>

Source: Brit. Petroleum
a. Express each country’s coal reserves in standard form. United States: 2,460,000,000,000,000 kg; Russia: 1,530,000,000,000,000 kg; India: 924,000,000,000,000 kg; Romania: 494,000,000,000,000 kg
b. How many times more coal does the United States have than Romania? \( 4.98 \times 10^3 \) or 498 times
c. One kilogram of coal has an energy density of 2.4 \times 10^{12} joules. What is the total energy density of the United States’ coal reserve? Express your answer in scientific notation. \( 5.90 \times 10^{23} \) joules
### 7-3 Enrichment

**Engineering Notation**

Engineering notation is a variation on scientific notation where numbers are expressed as powers of 1000 rather than as powers of 10. Engineering notation takes the familiar form of \( a \times 10^n \), but \( n \) is restricted to multiples of three and \( 1 \leq |a| < 1000 \).

One advantage to engineering notation is that numbers can be neatly expressed using SI prefixes. These prefixes are typically used for scientific measurements.

#### Table: Engineering Notation Prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Power of 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>peta</td>
<td>P</td>
<td>( 10^{15} )</td>
</tr>
<tr>
<td>tera</td>
<td>T</td>
<td>( 10^{12} )</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>( 10^{9} )</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>( 10^{6} )</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>( 10^{3} )</td>
</tr>
<tr>
<td>mil</td>
<td>m</td>
<td>( 10^{-3} )</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>( 10^{-6} )</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>( 10^{-9} )</td>
</tr>
<tr>
<td>femto</td>
<td>f</td>
<td>( 10^{-15} )</td>
</tr>
</tbody>
</table>

#### Example

**NUCLEAR POWER** The output of a nuclear power plant is measured to be 620,000,000 watts. Express this number in engineering notation and using SI prefixes.

To express a number in engineering notation, first convert the number to scientific notation.

**Step 1**

\[ 620,000,000 \rightarrow 6.2 \times 10^8 \] \( z = 6.2 \times 10^8 \)

**Step 2**

The decimal point moved 8 places to the left, so \( n = 8 \).

**Step 3**

\[ 620,000,000 = 6.2 \times 10^8 \times 10^4 = 6.2 \times 10^4 \]

Because 8 is not a multiple of 3, we need to round down \( n \) to the next multiple of 3.

**Step 4**

\[ 6.2 \times 10^8 = (6.2 \times 10^5) \times 10^3 = 6.2 \times 10^3 \]

Product of Powers

The output of the power plant is \( 620 \times 10^8 \) watts. Using the chart above, the prefix for \( 10^8 \) is found to be mega, or M. The output of the power plant is 620 megawatts, or 620 MW.

#### Exercises

**Express each number in engineering notation.**

1. \( 40,000,000,000 \times 10^7 \)
2. \( 180,000,000,000,000 \times 10^{12} \)
3. \( 60 \times 10^{-3} \)
4. \( 4.000 \times 10^{-6} \)

**Express each measurement using SI prefixes.**

5. 40,000,000,000 grams
6. 40,000,000 watts
7. 63,300,000,000 bytes
8. 0.0000002 meter
9. 63.1 terabytes (TB)
10. 200 nanometers (nm)

### 7-4 Study Guide and Intervention

#### Polynomials

**Degree of a Polynomial** A polynomial is a monomial or a sum of monomials. A **binomial** is the sum of two monomials, and a **trinomial** is the sum of three monomials. Polynomials with more than three terms have no special name. The degree of a monomial is the sum of the exponents of all its variables. The **degree of the polynomial** is the same as the degree of the monomial term with the highest degree.

#### Example

**Determine whether each expression is a polynomial. If so, identify the polynomial as a monomial, binomial, or trinomial. Then find the degree of the polynomial.**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Polynomial?</th>
<th>Monomial, Binomial, or Trinomial</th>
<th>Degree of the Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3x - 7yz )</td>
<td>Yes</td>
<td>binomial</td>
<td>3</td>
</tr>
<tr>
<td>-25</td>
<td>No</td>
<td>monomial</td>
<td>0</td>
</tr>
<tr>
<td>( 7x^3 + 3x^2 )</td>
<td>No</td>
<td>trinomial</td>
<td>3</td>
</tr>
</tbody>
</table>

#### Exercises

**Determine whether each expression is a polynomial. If so, identify the polynomial as a monomial, binomial, or trinomial.**

1. \( 36 \) yes; monomial
2. \( \frac{3}{4} + 5 \) no
3. \( 7x - x + 5 \) yes; binomial
4. \( 6x + 7x + 2 \) yes; trinomial
5. \( \frac{1}{4x^3} + 5y - 8 \) no
6. \( 6x + x^2 \) yes; binomial

**Find the degree of each polynomial.**

1. \( 4x^3y^2z \) 6
2. \( -2abc \) 3
3. 15m 1
4. \( r + 5t \) 1
5. \( 12.5 \) 0
6. \( 18x^2 + 4yz - 10y \) 2
7. \( 12.5x^2y^2 + 4x^2y^2 \) 5
8. \( -2x^3 + 7y^3 - 4y^3 \) 13
9. \( 4x^2 + 2y \) 9
10. \( 5.6 + bc \) 6
11. \( 4x^2 + 8x + 2 \) 2
12. \( 20.9abc + bc - n^3 \) 5
13. \( 21h^2m + 6h^2m^2 \) 7
**Skills Practice**

### Polynomials

**Write Polynomials in Standard Form**

Find the degree of each polynomial. Identify the leading coefficient.

1. \(2x^2 + 3x - 4\)
   - Degree: 2
   - Leading coefficient: 2

2. \(-5x^3 + 2x^2 - x + 3\)
   - Degree: 3
   - Leading coefficient: -5

3. \(-3x^4 + 4x^3 - 2x^2 + x - 1\)
   - Degree: 4
   - Leading coefficient: -3

4. \(x^5 - 2x^4 + 3x^3 - x^2 + x - 1\)
   - Degree: 5
   - Leading coefficient: 1

5. \(4x^6 - 3x^5 + 2x^4 - x^3 + x^2 - x + 1\)
   - Degree: 6
   - Leading coefficient: 4

6. \(-2x^7 + 3x^6 - 4x^5 + 5x^4 - 6x^3 + 7x^2 - 8x + 9\)
   - Degree: 7
   - Leading coefficient: -2

**Exercises**

Write each polynomial in standard form. Identify the leading coefficient.

1. \(3x^2 + 2x + 1\)
   - Leading coefficient: 3

2. \(-4x^3 + 5x^2 - 6x + 7\)
   - Leading coefficient: -4

3. \(9x^4 - 8x^3 + 7x^2 - 6x + 5\)
   - Leading coefficient: 9

4. \(-2x^5 + 3x^4 - 4x^3 + 5x^2 - 6x + 7\)
   - Leading coefficient: -2

5. \(x^6 - 2x^5 + 3x^4 - 4x^3 + 5x^2 - 6x + 7\)
   - Leading coefficient: 1

6. \(-3x^7 + 2x^6 - 1x^5 + 4x^4 - 5x^3 + 6x^2 - 7x + 8\)
   - Leading coefficient: -3
Chapter 7

7-4 Practice

Polynomials

Determine whether each expression is a polynomial. If so, identify the polynomial as a monomial, binomial, or trinomial.

1. \(7a^2b + 3b^2 - ab\) yes; binomial
2. \(\frac{1}{5}y^2 + y^2 - 9\) yes; binomial
3. \(6g^2h^2\) yes; monomial

Find the degree of each polynomial.

4. \(x + 3x^2 - 2x^3 + x^4\) \(4\)
5. \(3g^2h^2 + g^3h\) \(5\)
6. \(-2x^2y + 3xy^2 + x^2\) \(4\)
7. \(5x^2m - 2m^2 + n^2m^4 + n^2\) \(6\)
8. \(a^2bc + 2a^2c + b^2c^2\) \(6\)

Write each polynomial in standard form. Identify the leading coefficient.

9. \(8x^3 - 15 + 5x^4\) \(5\)
10. \(10x - 7 + x^4 + 4x^3\) \(x^4 + 4x^3 + 10x - 7; 1\)
11. \(13x^3 - 5 + 6x^2 - x\) \(6\)
12. \(4x + 2x^3 - 6x^2 + 2\) \(2x^3 - 6x^2 + 4x + 2; 2\)

GEOMETRY Write a polynomial to represent the area of each shaded region.

13. \[ab - b^2\]
14. \[a^2 - \frac{1}{4} \pi d^2\]

15. MONEY Write a polynomial to represent the value of \(t\) ten-dollar bills, \(f\) fifty-dollar bills, and \(h\) one-hundred-dollar bills. \(10t + 50f + 100h\)

16. GRAVITY The height above the ground of a ball thrown up with a velocity of 96 feet per second from a height of 6 feet is \(6 + 96t - 16t^2\) feet, where \(t\) is the time in seconds. According to this model, how high is the ball after 7 seconds? Explain.

\(-106\) ft; The height is negative because the model does not account for the ball hitting the ground when the height is 0 feet.

7-4 Word Problem Practice

Polynomials

1. PRIMES Mei is trying to list as many prime numbers as she can for a challenge problem for her math class. She finds that the polynomial expression \(n^2 - n + 41\) can be used to generate some, but not all, prime numbers. What is the degree of Mei's polynomial?

2. PHONE CALLS A long-distance telephone company charges a $19.95 standard monthly service fee plus $0.05 per minute of long-distance use. Write a polynomial to express the monthly cost of the phone plan if \(x\) minutes of long-distance use are used per month. What is the degree of the polynomial?

3. COSTUMES Jack’s mother is sewing the cape of his costume for a charity masked ball. The pattern for the cape (lying flat) is shown below. The radius of the neck hole is 6 inches. What is the area, in square feet, of the finished cape?

\(27.5\) ft\(^2\)

4. ARCHITECTURE Graphing the polynomial function \(y = -x^2 + 3\) produces an accurate drawing of the shape of an archway inside a historical library, where \(x\) is the horizontal distance in meters from the base of the arch and \(y\) is the height of the arch. At \(x = 0\), what is the height of the arch? \(3\) m

5. DRIVING A truck and a car leave an intersection. The truck travels south, and the car travels east. When the truck had gone 24 miles, the distance between the car and truck was four miles more than three times the distance traveled by the car heading east.

6. MONEY Write a polynomial to represent the value of \(r\) ten-dollar bills, \(t\) fifty-dollar bills, and \(h\) one-hundred-dollar bills. \(10r + 50t + 100h\)

7. GRAVITY The height above the ground of a ball thrown up with a velocity of 96 feet per second from a height of 6 feet is \(6 + 96t - 16t^2\) feet, where \(t\) is the time in seconds. According to this model, how high is the ball after 7 seconds? Explain.
Polynomial Functions
Suppose a linear equation such as \(-3x + y = 4\) is solved for \(y\). Then an equivalent equation, \(y = 3x + 4\), is found. Expressed in this way, \(y\) is a function of \(x\), or \(f(x) = 3x + 4\). Notice that the right side of the equation is a binomial of degree 1.

Higher-degree polynomials in \(x\) may also form functions. An example is \(f(x) = x^3 + 1\), which is a polynomial function of degree 3. You can graph this function using a table of ordered pairs, as shown at the right.

For each of the following polynomial functions, make a table of values for \(x\) and \(y = f(x)\). Then draw the graph on the grid.

1. \(f(x) = 1 - x^2\)
2. \(f(x) = x^2 - 5\)
3. \(f(x) = x^2 + 4x - 1\)
4. \(f(x) = x^2\)

### Example
An object is dropped from the top of a 179-foot cliff to the water below. The height of the object above the water can be modeled by \(h(t) = -16t^2 + 179\) where \(t\) is time in seconds.

a. Determine the height of the object after 0.5 second, 1 second, 1.5 seconds, and 2 seconds.

b. After how many seconds does the object hit the water? Round to the nearest hundredth.

### Exercises
1. An object is dropped from the top of a building that is 412 feet high. The distance, in feet, above the ground at \(x\) seconds is given by \(P(x) = -16x^2 + 412\).

   a. After how many seconds will the object be 100 feet above the ground? \(= 4.4\) s
   b. How many seconds will it take the object to reach the ground? \(= 5.1\) s

2. A bungee jumper free falls from the Royal Gorge suspension bridge over the Arkansas River, 1053 feet above the river. The height \(h\) of the bungee jumper above the river, in feet, after \(t\) seconds can be represented by \(h = -16t^2 + 1053\). Two seconds after the first bungee jumper falls, another person jumps down with an initial velocity of 80 feet per second.

   a. If the bungee cords are designed to stretch just enough so that the jumpers touch the water before springing back up, which jumper will touch the water first? How long does it take each jumper to touch the water? yes; 477 ft; 6 s, 4 s
Study Guide and Intervention

Adding and Subtracting Polynomials

Example 1
Find \((2x^2 + x - 8) + (3x - 4x^2 + 2)\).

Horizontal Method
Group like terms.
\((2x^2 + x) + (-4x^2 + 3x) + (-8 + 2)\)
\(-2x^2 + 4x - 6\).
The sum is \(-2x^2 + 4x - 6\).

Example 2
Find \((3x^2 + 5x) + (xy + 2x^3)\).

Vertical Method
Align like terms in columns and add.
\(3x^2 + 5x\)
\(+ xy + 2x^3\)
\(5x^3 + 6xy\)
The sum is \(5x^3 + 6xy\).

Exercises
Find each sum.

1. \((4x - 5) + (3x + 6)\)
   \(7x + 1\)

2. \((6x^2 + 9) + (4x^2 - 7)\)
   \(4x^2 + 6x + 2\)

3. \((6x + 2y + 6x) + (4xy - x)\)
   \(10xy + 5x + 2y\)

4. \((3x - 2p + 3) + (p^2 - 7 + p)\)
   \(4p^2 - 9p + 10\)

5. \((7x^2 + 2y) + (2p^2 + 8y + 1)\)
   \(2p^2 + 5p + 6y + 1\)

6. \((6x^2 + 3x) + (x^2 - 4x - 3)\)
   \(7x^2 - x \cdot 3\)

7. \((2x^2 - 4x + 4) - (2x - 5) - 6\)
   \(-5x + 5\)

8. \((2p - 5) + (3p + 6x) + (p - r)\)
   \(6p \)

9. \((5x^2 + 3x^2) + (x + 2x + 2) + (2 - 4)\)
   \(-4x + 8\)

Example 3
Find each difference.

1. \((2x^2 + x - 6) - (2x + x^2 + 3)\)
   \(3x^2 + 2x - 6\)

2. \((x^2 + 3x - 6) - (2x - x^2)\)
   \(-6x^2 - 3\)

3. \((3x^2 - 2x + 1) - (3x^2 + 2x + 3)\)
   \(-2x^2 - 9\)

4. \((3x^2 + 2x + 3) - (2x + x^2)\)
   \(2x^2 - 9\)

5. \((3x^2 + 2x + 3) - (2x + x^2)\)
   \(2x^2 - 9\)

Example 4
Find \((3x^2 + 2x - 6) - (2x + x^2 + 3)\).

Horizontal Method
Use additive inverses to rewrite as addition.
\((3x^2 + 2x - 6) + (2x + x^2 + 3)\)
\(4x^2 + 4x - 3\)

7. \((4x - 7) - (2x - 4) - (2x + 6 - 3)\)
   \(6\)

8. \((3x - 2x) - (3x^2 + 5x - 1)\)
   \(-7x + 1\)

9. \((2x^2 - 6) + (5x^2 + 2) + (3x^2 - 7)\)
   \(6x^2 + 11\)

10. \((5x^2 + 4x + 2 - 4x + 2) + (2x^2 - 8)\)
    \(7x^2 + 7x\)

11. \((x^2 + 3x + 1) - (x^2 - 2)\)
    \(x + 1\)

12. \((3x^2 + 5x + 5) - (2x^2 - 8)\)
    \(x^2 + 8x + 13\)

13. \((2x^2 + 3x + 5) - (3x^2 - 5x + 2)\)
    \(2x^2 + 3x + 2\)

14. \((2x^2 - 5x + 2) - (2x^2 - 8)\)
    \(2x^2 + 8\)

15. \((9x^2 + 3x - 2) - (2x^2 + x - 1)\)
    \(5x^2 + 4x + 2\)

16. \((6x^2 + 3x + 1) - (7x^2 - 2x + 4)\)
    \(13x^2 - 3x - 3\)
Find each sum or difference.

1. \((2x + 3y) + (4x + 9y) = 6x + 12y\)
2. \((6m + 5n) + (4t + 8s) = 14s + 9t\)
3. \((5a + 9b) - (2a + 4b) = 3a + 5b\)
4. \((11m - 7n) - (2m + 6n) = 9m - 13n\)
5. \((m^2 - m) + (2m + m^3) = 2m^2 + m\)
6. \((x^3 - 3x) - (2x^2 + 5x) = -x^2 - 8x\)
7. \((d^2 - d + 5) - (2d + 5) = d^2 - 3d\)
8. \((2h^2 - 5h) + (7h - 3h^3) = -h^3 + 2h\)
9. \((5f + g - 2) + (-2f + 3) = 3f + g + 1\)
10. \((6b^2 + 2h + 9) + (4k - 5h) = 10k^2 - 3k + 9\)
11. \((x^2 - x + 1) - (3x - 1) = x^2 - 4x + 2\)
12. \((b^3 + ab - 2) - (2b^2 + 2ab) = -b^2 - ab - 2\)
13. \((7a^2 + 4z - z) - (-5 + 3x^2) = 4z^2 - z + 9\)
14. \((5 + 4n + 2t) + (-6t - 8) = -3n + 4t\)
15. \((4f + 2d) + (-4 + 2t) = 4f^2 + 2t - 2\)
16. \((3y^2 + 7g) - (4g + 8y^2) = -9g + 3g\)
17. \((2a^2 + 8a + 4) - (a^2 - 3) = a^2 + 8a + 7\)
18. \((3c^2 - 7x + 5) - (-x^2 + 4c) = 2x^2 + 11x + 5\)
19. \((7x^2 + y + 1) - (4y + 3y^2 - 3) = 4x^2 + 5y + 4\)
20. \((2c^2 + 7c + 4) + (c^2 + 1 - 9c) = 11c - 5\)
21. \((a^2 + 3n + 2) - (2c^2 - 6n - 2) = -n^2 + 9n + 4\)
22. \((a^3 + ab - 2b^3) + (b^2 + 4a^3 - ab) = 5a^2 - 2b^2\)
23. \((e^2 - 5^2 - 6) + (2c^2 + 5 + f) = 3l^2 - 4f - 1\)
24. \((2m^2 + 5m + 1) - (4m^3 - 3m - 3) = -2m^2 + 8m + 4\)
25. \((2x - 6x + 2) + (-5x^2 + 7x - 4) = 6x^2 - 13x + 6\)
26. \((5b^2 - 9b - 5) + (b^2 + 6 + 2b) = 6b^2 - 7b + 11\)
27. \((2x^3 - 5x - 2) + (x^2 - 4x) + (3c^2 + x + 5) = 6x^3 - x + 3\)

Answers

Find each sum or difference.

1. \((4p + 5) + (-3y - 1) = -3p^2 + 4\)
2. \((x^3 + 3x) - (5x + 2x^2) = -3x^2 - 2x\)
3. \((6h^2 + 8e + 2) - (2h + 3) = 4k^2 + 6k - 1\)
4. \((3m^2 + 6m) + (m^2 - 5m + 7) = 3m^2 + m + 7\)
5. \((5a^2 + 6a + 2) - (7a^2 - 7a + 5) = -2a^2 + 13a - 3\)
6. \((4p^2 + p + 9) + (p^2 + 3p - 1) = -3p^2 + 2p + 8\)
7. \((x^2 + 3x + 1) - (x^2 + 7 - 12c) = 9x - 6\)
8. \((6c^2 - x + 1) - (-4 + 2c^2 + 8x) = 4x^2 - 9x + 5\)
9. \((4p^2 + 2y - 8) - (7y^2 + 4 - y) = -3y^2 + 3y - 12\)
10. \((2m^2 - 4u - 1) + (-5 + 5u^3 - 3u) = 6m^2 - 7u - 6\)
11. \((5m^2 - 2a - 3) + (3a^3 - u + 4) = 7u^2 - 3u + 1\)
12. \((6x^2 - 3d + 5) + (5d^2 - 2 - d) = 9d^2 + d\)
13. \((3c^2 - 7) - (4k + 8h^2 + 1) = -5h^2 + 3h - 2\)
14. \((3l^2 + 4x - 6) - (-5x^2 - y^2 - 5) = -4x^2 + 2y^2 - 1\)
15. \((5f^2 + 2t^2) = 4k^2 + 6t - (-4k + h^2 - 3) = 8t^2 - 31 - 5\)
16. \((2x + 6y - 3m + 4x + 6z - 8y) + (x - 3y + z) = 7x - 5y + 4z\)
17. \((3f^2 - 7f - 3) - (5f^2 - 2 - 1) - (2f^2 - 3 + f) = f^2 - 10f + 1\)
18. \((5f^2 - 2 - 1) - (2f^2 - 3 + f) = f^2 - 10f + 1\)
19. \((3f^2 - 7f - 3) - (5f^2 - 2 - 1) - (2f^2 - 3 + f) = f^2 - 10f + 1\)
20. \((3f^2 - 7f - 3) - (5f^2 - 2 - 1) - (2f^2 - 3 + f) = f^2 - 10f + 1\)

23. BUSINESS The polynomial \(s^2 - 70s^2 + 1500s - 10,800\) models the profit a company makes on selling an item at a price \(s\). A second item sold at the same price brings in a profit of \(s^2 - 50s^2 + 455s - 500\). Write a polynomial that expresses the total profit from the sale of both items. \(2s^2 - 100s^2 + 1950s - 15,800\)

24. GEOMETRY The measures of two sides of a triangle are given. If \(P\) is the perimeter, and \(P = 10x + 5y\), find the measure of the third side. \(2x + 2y\)
7-5 Word Problem Practice

Adding and Subtracting Polynomials

1. BUILDING Find the simplest expression for the perimeter of the triangular roof truss. 
   \[5a + 6a - 3\]

2. GEOMETRY Write a polynomial to show the area of the large square below.
   \[a^2 + 2ab + b^2\]

3. FIREWORKS Two bottle rockets are launched straight up into the air. The height, in feet, of each rocket at \(t\) seconds after launch is given by the polynomial equations below. Write an equation to show how much higher Rocket A traveled. Rocket A: \(H_A = -16t^2 + 122t\) Rocket B: \(H_B = -16t^2 + 84t\)

4. ENVELOPES An office supply company produces yellow document envelopes. The envelopes come in a variety of sizes, but the length is always 4 centimeters more than double the width. Write a polynomial expression to give the perimeter of any of the envelopes.
   \[6x + 8\]

5. INDUSTRY Two identical right cylindrical steel drums containing oil need to be covered with a fire-resistant sealant. In order to determine how much sealant to purchase, George must find the surface area of the two drums. The surface area (including the top and bottom bases) is given by the following formula.
   \[S = 2\pi rh + 2\pi r^2\]

   a. Write a polynomial to represent the total surface area of the two drums.
   \[4\pi rh + 4\pi r^2\]

   b. Find the total surface area if the height of each drum is 2 meters and the radius of each is 0.5 meters. Let \(\pi = 3.14\).

   c. The fire resistant sealant must be applied while they are stacked vertically in groups of three. If \(h\) is the height of each drum and \(r\) is the radius, write a polynomial to represent the exposed surface area.
   \[6\pi rh + \pi r^2\]

Word Problem Practice

7-5 Enrichment

Circular Areas and Volumes

Area of Circle  Volume of Cylinder  Volume of Cone

\[A = \pi r^2 \quad V = \pi rh \quad V = \frac{1}{3} \pi r^2 h\]

Write an algebraic expression for each shaded area. (Recall that the diameter of a circle is twice its radius.)

1. \(\pi x^2\) - \(\pi \left(\frac{x}{2}\right)^2\) = \(\pi x^2\)

2. \(\frac{\pi}{2} (y^2 + 2xy)\)

3. \(\frac{19}{2} \pi x^2\)

Write an algebraic expression for the total volume of each figure.

4. \(\frac{2}{3} \pi x^0\)

5. \(\frac{\pi}{12} [13x^2 + (4a + 9b)x]\)

Each figure has a cylindrical hole with a radius of 2 inches and a height of 5 inches. Find each volume.

6. \(\frac{175\pi}{4} x^3 - 20\pi \text{ in}^5\)

7. \(3\pi x^3 - 20\pi \text{ in}^5\)
### Exercises

**Find each product.**

1. \(x(5x + x^2)\)
   - \(5x^2 + x^3\)
2. \((x + 2)(3x + 2)\)
   - \(3x^2 + 10x + 4\)
3. \((-2x + 3y)(y + 2)\)
   - \(-2xy + 5y^2 - 4xy - 8x^2\)
4. \(-2g^2(4g - 2g)\)
   - \(-9g^2 + 4g^2\)
5. \(3x^2 + x^3 + x^2\)
   - \(5x^3 + 3x^3 + 3x^2\)
6. \((-2g - 2h)(5g - 4)\)
   - \(8g^2 - 12x - 12ax^2 - 12ax^2\)
7. \(3x^3 + 2x^2 - 3x - 2\)
   - \(3x^2 - 5x - 11\)

**Simplify each expression.**

10. \(-5x + 3\)
   - \(3x^2 - 9x\)
11. \(-3x^2 + 5\)
   - \(-2x^2 + 2x\)
12. \(-6x^2 + 7x - 2 - 4x + 5\)
   - \(4a + 4b\)
13. \(4x^2 - 3x + 2\)
   - \(32x^2 + 68x^2\)
14. \(4(3x^2 + n - 8)\)
   - \(-28(3n + 2x + 1)\)
15. \(-2x^2 + 3x - 2\)
   - \(-5x - 3\)
16. \(-2x^2 + 3x - 2\)
   - \(-2x^2 + 6x - 12\)

**Example 5** Find \(-3x^3(4x^2 + 6x - 8)\).

**Horizontal Method**

\[\begin{align*}
-3x^4 & - 18x^3 + 24x^2 \\
-3x^4 & + 6x^2 + 8x \\
-12x^3 & + 18x^2 + 24x \\
\end{align*}\]

The product is \(-12x^3 - 18x^2 + 24x^2\).
7-6 Skills Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. \(6q + 13\)  
2. \(-4c + 4\)

Simplify each expression.

13. \(3w^2 + 7w + 4w^2 + 3w - 5p + 3p\)
14. \(5f^2 - 3f - 2f + 5f\)

Solve each equation.

21. \(3a + 2 + 5 = 2a + 4\)  
22. \(24x + 2 - 8 = 4(x + 3)\)

19. \(4b - 5b - 3b^2 - 7b - 4\)  
20. \(3m(3m + 6) - 3m^2 + 4m + 1\)

23. \(5y + 1 + 2\)  
24. \(2b + 6\)

25. \(6m - 2\)  
26. \(3c + 5\)  

7-6 Practice

Multiplying a Polynomial by a Monomial

Find each product.

1. \(2(-7h^2 - 4h)\)  
2. \(6pq(3p^3 + 4q)\)
3. \(5k(3k + 2k)\)  
4. \(-3(k - 2j^2 + 3)\)

Simplify each expression.

7. \(-2t(3t - 4) + 7t\)  
8. \(-t^2 + 5\)
9. \(-6t(3t - 3) + 5(2t^2 + 9t - 3)\)  
10. \(-3m^3 + 5m + 6 + 3m(2m^2 + 3m + 1)\)

Solve each equation.

12. \(5x + 2 + 3\)  
13. \(3(2a + 2) + 5 = 2(2a - 2) - 3\)
14. \(4x + 3 + 2 = 2nx + 8\)  
15. \(8(x + 1) = 4(3x + 1) - 9 - \frac{1}{2}\)
16. \(t(4 - 1) + 1 = t + 2 - 1\)  
17. \(a(a - 5) + 6 = a(3 + 2) - 4 - 4\)

18. NUMBER THEORY Let \(x\) be an integer. What is the product of twice the integer added to three times the next consecutive integer? \(5x + 3\)

19. INVESTMENTS Kent invested \$5000 in a retirement plan. He allocated \(x\) dollars of the money to a bond account that earns 4% interest per year and the rest to a traditional account that earns 5% interest per year.

a. Write an expression that represents the amount of money invested in the traditional account. \(5000 - x\)

b. Write a polynomial model in simplest form for the total amount of money \(T\) Kent has invested after one year. \((\text{Hint: Each account has } A + 1A, \text{ where } A \text{ is the original amount in the account and } I \text{ is its interest rate.}) T = 5250 - 0.01x\)

c. If Kent put \$500 in the bond account, how much money does he have in his retirement plan after one year? \$5245

Answers (Lesson 7-6)
1. **NUMBER THEORY** The sum of the first \( n \) whole numbers is given by the expression \( \frac{1}{2}(n^2 + n) \). Expand the equation by multiplying, then find the sum of the first 12 whole numbers. 
\[
\frac{n^2 + n}{2} = 78
\]

2. **COLLEGE** Troy's boss gave him $700 to start his college savings account. Troy's boss also gives him $40 each month to add to the account. Troy's mother gives him $80 each month, but has been doing so for 4 fewer months than Troy's boss. Write a simplified expression for the amount of money Troy has received from his boss and mother after \( m \) months. 
\[
90m + 500
\]

3. **LANDMARKS** A circle of 50 flags surrounds the Washington Monument. Suppose a new sidewalk 12 feet wide is installed just around the outside of the circle of flags. The outside circumference of the sidewalk is 1.10 times the circumference of the circle of flags. Write an equation that equates the outside circumference of the sidewalk to 1.10 times the circumference of the circle of flags. Solve the equation for the radius of the circle of flags. 
\[
1.10(2\pi r) = 2\pi(r + 12); r = 120 \text{ ft}
\]

4. **MARKET** Sophia went to the farmers' market to purchase some vegetables. She bought peppers and potatoes. The peppers were $0.39 each and the potatoes were $0.29 each. She spent $3.88 on vegetables, and bought 4 more potatoes than peppers. If \( x \) is the number of peppers, write and solve an equation to find out how many of each vegetable Sophia bought. 
\[
3.88 = x(0.39) + (x + 4)(0.29)
\]

5. **GEOMETRY** Some monuments are constructed as rectangular pyramids. The volume of a pyramid can be found by multiplying the area of its base \( B \) by one third of its height. The area of the rectangular base of a monument in a local park is given by the polynomial equation \( B = x^2 - 4x - 12 \). 
\[
V = \frac{1}{3}x^2 - \frac{40}{3}x - 40
\]

6. Evaluate the product in Exercise 5 for values of \( x \) from 1 through 5. On another sheet of paper, make drawings to show why these numbers are called the triangular numbers. 
\[
1, 3, 6, 10, 15
\]

7. **Figurate Numbers** The numbers below are called **pentagonal numbers**. They are the numbers of dots or disks that can be arranged as pentagons. 

![Pentagonal Numbers](image)

1. Find the product \( \frac{1}{2}n(3n - 1) \). 
2. Evaluate the product in Exercise 1 for values of \( n \) from 1 through 4. 
3. What do you notice? **They are the first four pentagonal numbers.**
4. Find the next six pentagonal numbers. 
5. Find the product \( \frac{1}{2}n(n + 1) \). 
6. Evaluate the product in Exercise 5 for values of \( n \) from 1 through 5. 
7. Find the product \( n(2n - 1) \). 
8. Evaluate the product in Exercise 7 for values of \( n \) from 1 through 5. Draw these hexagonal numbers. 
9. Find the first 5 square numbers. Also, write the general expression for any square number. 
10. If you pile 10 oranges into a pyramid with a triangle as a base, you get one of the tetrahedral numbers. How many layers are there in the pyramid? How many oranges are there in the bottom layers? 
11. Evaluate the expression \( \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n \) for values of \( n \) from 1 through 5 to find the first five tetrahedral numbers.
### Chapter 7

#### 7-7 Study Guide and Intervention

**Multiplying Polynomials**

**Multiply Binomials** To multiply two binomials, you can apply the Distributive Property twice. A useful way to keep track of terms in the product is to use the FOIL method as illustrated in Example 2.

**Example 1** Find \((x + 3)(x - 4)\).

**Horizontal Method**

\[ (x + 3)(x - 4) \]

\[ = x(x) + x(-4) + 3(x) + 3(-4) \]

\[ = x^2 - 4x + 3x - 12 \]

\[ = x^2 - x - 12 \]

**Vertical Method**

\[
\begin{align*}
x + 3 \\
\hline
x & - 4 \\
\hline
x^2 & - 4x \\
+ & 3x \\
\hline
x^2 & - x - 12
\end{align*}
\]

The product is \(x^2 - x - 12\).

**Example 2** Find \((x - 2)(x + 5)\) using the FOIL method.

**First**

\[ (x - 2)(x + 5) \]

**Out**

\[ = (x)(x) + (x)(5) = x^2 + 5x \]

**Inner**

\[ = (-2)(x) + (-2)(5) = -2x - 10 \]

**Last**

\[ = (-2)(5) = -10 \]

The product is \(x^2 + 3x - 10\).

### Exercises

#### Find each product.

1. \((x + 2)(x + 3)\)
   \[x^2 + 5x + 6\]

2. \((x - 2)(x - 3)\)
   \[x^2 - 5x + 6\]

3. \((x + 3)(x - 2)\)
   \[x^2 + x - 6\]

4. \((x - 3)(x + 1)\)
   \[x^2 - 2x - 3\]

5. \((x + 2)(x - 3)\)
   \[x^2 + x - 6\]

6. \((x - 3)(x + 2)\)
   \[x^2 + x - 6\]

7. \((x + 3)(x - 2)\)
   \[x^2 + x - 6\]

8. \((x - 2)(x + 3)\)
   \[x^2 + x - 6\]

9. \((x + 2)(x - 3)\)
   \[x^2 + x - 6\]

10. \((x - 3)(x + 2)\)
    \[x^2 + x - 6\]

11. \((x + 2)(x - 3)\)
    \[x^2 + x - 6\]

12. \((x - 3)(x + 2)\)
    \[x^2 + x - 6\]

13. \((x + 3)(x - 2)\)
    \[x^2 + x - 6\]

14. \((x - 2)(x + 3)\)
    \[x^2 + x - 6\]
### 7-7 Skills Practice

**Multiplying Polynomials**

Find each product.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>((m + 4)(m + 1))</td>
<td>(m^2 + 5m + 4)</td>
</tr>
<tr>
<td>((x + 2)(x + 2))</td>
<td>(x^2 + 4x + 4)</td>
</tr>
<tr>
<td>((b + 3)(b + 4))</td>
<td>(b^2 + 7b + 12)</td>
</tr>
<tr>
<td>((t + 4)(t - 3))</td>
<td>(t^2 + t - 12)</td>
</tr>
<tr>
<td>((r + 1)(r - 2))</td>
<td>(r^2 - r - 2)</td>
</tr>
<tr>
<td>((2x - 6)(x + 3))</td>
<td>(2x^2 - 18)</td>
</tr>
<tr>
<td>((2x + 5)(x - 4))</td>
<td>(2x^2 - 3x - 20)</td>
</tr>
<tr>
<td>((3n - 7)(n + 3))</td>
<td>(3n^2 + 2n - 21)</td>
</tr>
<tr>
<td>((q + 5)(5q - 1))</td>
<td>(5q^2 + 24q - 5)</td>
</tr>
<tr>
<td>((m + 2)(m - 3))</td>
<td>(m^2 - m - 6)</td>
</tr>
<tr>
<td>((5a - 2)(2a - 3))</td>
<td>(10a^2 - 19a + 6)</td>
</tr>
<tr>
<td>((4h - 2)(4h - 1))</td>
<td>(16h^2 - 12h + 2)</td>
</tr>
<tr>
<td>((x - y)(2x - y))</td>
<td>(2x^2 - 3xy + y^2)</td>
</tr>
<tr>
<td>((a + 2)(a + 3))</td>
<td>(a^2 + 5a + 6)</td>
</tr>
<tr>
<td>((m + 3)(m + 3))</td>
<td>(m^2 + 6m + 9)</td>
</tr>
</tbody>
</table>

### 7-7 Practice

**Multiplying Polynomials**

Find each product.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>((q + 6)(q - 5))</td>
<td>(q^2 + 11q + 30)</td>
</tr>
<tr>
<td>((n - 4)(n - 6))</td>
<td>(n^2 - 10n + 24)</td>
</tr>
<tr>
<td>((b + 6)(b - 4))</td>
<td>(6b^2 - 10b - 24)</td>
</tr>
<tr>
<td>((a - 2)(a - 6))</td>
<td>(a^2 - 8a + 12)</td>
</tr>
<tr>
<td>((x + 8)(x - 3))</td>
<td>(x^2 + 5x - 24)</td>
</tr>
<tr>
<td>((x + 9)(2x - 4))</td>
<td>(4x^2 - 10x - 36)</td>
</tr>
<tr>
<td>((x + 5)(2x - 4))</td>
<td>(4x^2 - 18x + 8)</td>
</tr>
<tr>
<td>((3x - b)(2a - b))</td>
<td>(6a^2 - 5ab + b^2)</td>
</tr>
<tr>
<td>((5x + 3)(x - 6))</td>
<td>(5x^2 + 9x - 18)</td>
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<td>(6x^2 + 7x + 6)</td>
</tr>
<tr>
<td>((x + 5)(2x - 3))</td>
<td>(4x^2 - 12x + 8x + 6x - 9)</td>
</tr>
</tbody>
</table>

**GEOMETRY** Write an expression to represent the area of each figure.

21. \(4x^2 - 2x - 2\) units$^2$
22. \(5x - 4\) units$^2$
23. **NUMBER THEORY** Let \(x\) be an even integer. What is the product of the next two consecutive even integers? \(x^2 + 6x + 8\)
24. **GEOMETRY** The volume of a rectangular pyramid is one third the product of the area of its base and its height. Find an expression for the volume of a rectangular pyramid whose base has an area of \(3x^2 + 12x + 9\) square feet and whose height is \(x + 3\) feet. \(x^3 + 7x^2 + 15x + 9\) ft$^3$
7-7 Word Problem Practice

Multiplying Polynomials

1. THEATER The Loft Theater has a center seating section with 3c + 8 rows and 4c – 1 seats in each row. Write an expression for the total number of seats in the center section. $12c^2 + 29c - 8$

2. CRAFTS Suppose a quilt made up of squares has a length-to-width ratio of 5 to 4. The length of the quilt is 5 inches. The quilt can be made slightly larger by adding a border of 1-inch squares all the way around the perimeter of the quilt. Write a polynomial expression for the area of the larger quilt.

3. SERVICE A folded United States flag is sometimes presented to individuals in recognition of outstanding service to the country. The flag is presented folded in a triangle. Often the recipient purchases a case designed to display the folded flag to protect it from wear. One such display case has dimensions (in inches) shown below. Write a polynomial expression that represents the area of wall space covered by the display case.

4. MATH FUN Think of a whole number. Subtract 2. Write down this number. Take the original number and add 2. Write down this number. Find the product of the numbers you wrote down. Subtract the square of the original number. The result is always –4. Use polynomials to show how this number trick works.

5. ART The museum where Julia works plans to have a large wall mural replica of Vincent van Gogh’s The Starry Night painted in its lobby. First, Julia wants to paint a large frame around where the mural will be. The mural’s length will be 5 feet longer than its width, and the frame will be 2 feet wide on all sides. Julia has only enough paint to cover 100 square feet of wall surface. How large can the mural be?

- Write an expression for the area of the mural.
- Write an expression for the area of the frame.
- Write and solve an equation to find how large the mural can be.

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- Write an expression for the area of the mural.
- Write an expression for the area of the frame.
- Write and solve an equation to find how large the mural can be.

8. Enrichment

Pascal’s Triangle

This arrangement of numbers is called Pascal’s Triangle. It was first published in 1665, but was known hundreds of years earlier.

1. Each number in the triangle is found by adding two numbers. What two numbers were added to get the 6 in the 5th row? 1 4 6 4 1

2. Describe how to create the 6th row of Pascal’s Triangle.

3. Write the numbers for rows 6 through 10 of the triangle.

4. Multiply to find the expanded form of each product.

5. Use Pascal’s Triangle to write the expanded form of $(a + b)^5$.
**7-7 Spreadsheet Activity**

**Multiplying Polynomials**

**Example**

A box is made by cutting a square with sides \(x\) inches long from each corner of a piece of cardboard and folding up the sides. If the piece of cardboard is 15 inches long and 12 inches wide, what integer value of \(x\) allows you to make the box with the greatest volume? What is the volume?

**Step 1**
The finished box will be \(x\) inches high, \(12 - 2x\) inches wide, and \(15 - 2x\) inches long. The volume of the box is \(x(12 - 2x)(15 - 2x)\) cubic inches.

**Step 2**
Use Column A of the spreadsheet for the value of \(x\). Enter the formulas for the width, length, and volume in Columns C, D, and E.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**Exercises**

Use the spreadsheet to find the value of \(x\) that allows the box with the greatest volume for each piece of cardboard. State the volume of the box.

1. 16 inches long and 10 inches wide
   
   2. 24 inches long and 18 inches wide
   
   3. 144 in\(^3\)
   
   4. 648 in\(^3\)
   
   5. 32 inches long and 16 inches wide
   
   6. 48 inches long and 24 inches wide
   
   7. 260 in\(^3\)
   
   8. 8192 in\(^3\)

**7.** Study the spreadsheet you created for Exercise 5. Suppose \(y\) is the volume of the box with a height of \(x\) inches. If you were to graph the ordered pairs \((x, y)\) and connect them with a smooth curve, what would you expect the graph to look like? Use the graphing tool in the spreadsheet to verify your conjecture. **Sample answer:** The graph would rise from \((0, 0)\) to the point where \(x\) gives the greatest volume and then fall back down toward the \(x\)-axis.

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**7-8 Study Guide and Intervention**

**Special Products**

**Squares of Sums and Differences**

Some pairs of binomials have products that follow specific patterns. One such pattern is called the square of a sum. Another is called the square of a difference.

<table>
<thead>
<tr>
<th>Square of a sum</th>
<th>Square of a difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a + b)^2 = a^2 + 2ab + b^2)</td>
<td>((a - b)^2 = a^2 - 2ab + b^2)</td>
</tr>
</tbody>
</table>

**Example 1**

Find \((3a + 4)(3a - 4)\).

**Example 2**

Find \((2x - 9)(2x + 9)\).

**Exercises**

Find each product.

1. \((x - 6)^2\)
2. \((3p + 4)^2\)
3. \((4x - 5)^2\)

4. \((x - 2)^2\)
5. \((2x + 3)^2\)
6. \((m + 5)^2\)

7. \((a + 3)^2\)
8. \((3 - p)^2\)
9. \((a - 5y)^2\)

10. \((y + 4)^2\)
11. \((3 + x)^2\)
12. \((3x - 2)^2\)

13. \((x - 8)^2\)
14. \((x^2 + 1)^2\)
15. \((m^2 - 2)^2\)

16. \((x^2 - 1)^2\)
17. \((2x^2 - 3)^2\)
18. \((\frac{3}{4} + x)^2\)

19. \((x - 4)^2\)
20. \((2p + 4)^2\)
21. \((\frac{3}{2} - 2)^2\)

22. \((x^2 - 8x^2 + 16)^2\)
23. \((4p^2 + 16p + 16)^2\)
24. \((\frac{4}{9}x^2 - \frac{8}{3}x + 4\))
Skills Practice

Product of a Sum and a Difference
There is also a pattern for the product of a sum and a difference of the same two terms, \((a + b)(a - b)\). The product is called the difference of squares.

**Product of a Sum and a Difference**

\[(a + b)(a - b) = a^2 - b^2\]

**Example**

Find \((5x + 3y)(5x - 3y)\).

\[(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2 = 25x^2 - 9y^2\]

The product is \(25x^2 - 9y^2\).

**Exercises**

Find each product.

1. \((x - 4)(x + 4)\)
2. \((p + 2)(p - 2)\)
3. \((4x - 5)(4x + 5)\)
4. \((2x - 1)(2x + 1)\)
5. \((h + 7)(h - 7)\)
6. \((m - 5)(m + 5)\)
7. \((2d - 3)(2d + 3)\)
8. \((3x - 5)(3x + 5)\)
9. \((a - y)(a + y)\)
10. \((-4xy + 4x)\)
11. \((8 + 4x)(8 - 4x)\)
12. \((3a - 2b)(3a + 2b)\)
13. \((3y - 8)(3y + 8)\)
14. \((x^2 - 12x + 1)\)
15. \((m^2 - 5)(m^2 + 5)\)
16. \((x^2 - 2x + 2)\)
17. \((h^2 - k^2)(h^2 + k^2)\)
18. \(\left(\frac{1}{4}x^2 + 2\right)^2\)
19. \((3x - 2y^3)(3x + 2y^3)\)
20. \((2p - 5q)(2p + 5q)\)
21. \((3x^2 - 2y^4)^2\)
22. \((4p^2 - 25r^2)^2\)
23. \((9x^2 - 4y^4)\)

**GEOMETRY**

The length of a rectangle is the sum of two whole numbers. The width of the rectangle is the difference of the same two whole numbers. Using these facts, write a verbal expression for the area of the rectangle. The area is the square of the larger number minus the square of the smaller number.
7-8 Practice

Special Products

Find each product.

1. \((a + 9b)^2\)
2. \((b + 8c)^2\)
3. \((x - 10)^2\)
4. \((r - 11)^2\)
5. \((p + 7)^2\)
6. \((b + 6)^2\)
7. \((x + 3)^2\)
8. \((4x + 2)^2\)
9. \((5a - 4)^2\)
10. \((6b + 1)^2\)
11. \((3c + 4)^2\)
12. \((7a - 2)^2\)
13. \((5b + 3)^2\)
14. \((4d - 7)^2\)
15. \((3g + 9)^2\)
16. \((4k + 5)^2\)
17. \((a + 6a)^2\)
18. \((a - 7p)^2\)
19. \((6h - m)^2\)
20. \((k - 6y)^2\)
21. \((a - 12u)^2\)
22. \((b - 12k)^2\)
23. \((3n - 2b)^2\)
24. \((36n - 49b^2\)
25. \((6a - 7b)^2\)
26. \((6a + 4g)^2\)
27. \((9b + 2y)^2\)
28. \((5p - 2m)^2\)
29. \((8p - 12p^2)^2\)
30. \((9p + 4g)^2\)
31. \((-5b - 2g)^2\)
32. \((2b + 3g + 4)^2\)
33. \((3b - 4b - g)^2\)
34. \((2b + 3g + 4)^2\)
35. \((5p - 2m)^2\)
36. \((6a - 7b)^2\)

7-8 Word Problem Practice

Special Products

1. PROBABILITY The spinner below is divided into 2 equal sections. If you spin the spinner 2 times in a row, the possible outcomes are shown in the table below.

<table>
<thead>
<tr>
<th>Red</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>60%</td>
<td>40%</td>
</tr>
</tbody>
</table>

What is the probability of spinning a red and a blue in two spins? 50%

2. GRAVITY The height of a penny \(t\) seconds after being dropped down a well is given by the product of \((10 - 4t)\) and \((10 + 4t)\). Find the product and simplify. What type of special product does this represent? 100 - 16\(t^2\); product of a sum and difference

3. TRAFFIC PLANNING The Lincoln Memorial in Washington, D.C., is surrounded by a circular drive called Lincoln Circle. Suppose the National Park Service wants to change the layout of Lincoln Circle so that there are two concentric circular roads. Write a polynomial equation for the area \(A\) of the space between the roads if the radius of the smaller road is 10 meters less than the radius of the larger road.

\[ A = 20\pi r^2 - 100\pi \]

4. BUSINESS The Combo Lock Company finds that its profit data from 2005 to the present can be modeled by the function \(y = 4e^t + 44e^{-6t} + 121\), where \(y\) is the profit \(n\) years since 2005. Which special product does this polynomial demonstrate? Explain.

5. STORAGE A cylindrical tank is placed along a wall. A cylindrical PVC pipe will be hidden in the corner behind the tank. See the side view diagram below. The radius of the tank is \(r\) inches and the radius of the PVC pipe is \(s\) inches.

\[ \pi \cdot r \cdot s \]

a. Use the Pythagorean Theorem to write an equation for the relationship between the two radii. Simplify your equation so that there is a zero on one side of the equals sign.

\[ 0 = r^2 - 6rs + 5s^2 \]

b. Write a polynomial equation you could solve to find the radius \(s\) of the PVC pipe if the radius of the tank is 20 inches.

\[ 0 = s^2 - 120s + 400 \]
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OFFENDING COMMAND:

STACK: